

Zero-Inflation Meta-analysis
ANOVA Repeated measure model Multiple regression
Mixed model Animal model
Zero-Inflation ANCOVA Split-Plot Survival analysis
Random Regression Phylogenetic mixed model
Ridge Regression MANCOVA Censoring
Threshold model GLM Nested ANOVA
Over-dispersion Logistic Regression

Generalised Linear Mixed Models

GLMM

Generalised Linear Mixed Models

GLMM

MCMCglmm: an R package for fitting Bayesian GLMM
using Markov chain Monte Carlo

A Toy Example

```
y = rnorm(5, mean = 0, sd = sqrt(1))
```

```
0.256 -1.995 -0.362 0.685 0.118
```

A Toy Example

```
y = rnorm(5, mean = 0, sd = sqrt(1))
```

```
0.256 -1.995 -0.362 0.685 0.118
```

What is the mean and variance of the distribution
these numbers were drawn from?

Likelihood

$$\Pr(y \mid \text{mean, variance})$$

μ

Probability of the data *given* the parameters

σ^2

0.256

-1.995

-0.362

0.685

0.118

Likelihood

$$\Pr(y \mid \text{mean, variance})$$

0.256

μ

Probability of the data *given* the parameters

-1.995

σ^2

Bayesian

$$\Pr(\text{mean, variance} \mid y)$$

-0.362

0.685

Probability of the parameters *given* the data

0.118

Likelihood

```
prod(dnorm(y, mean=0, sd=sqrt(1)))  
0.00098
```

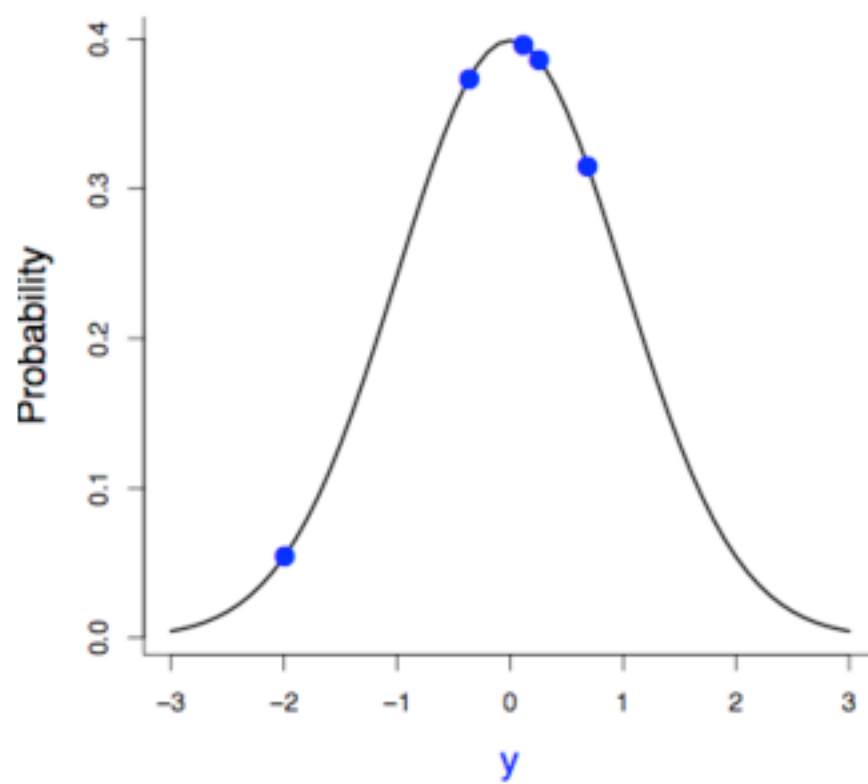
0.256

μ

-1.995

σ^2

-0.362



0.685

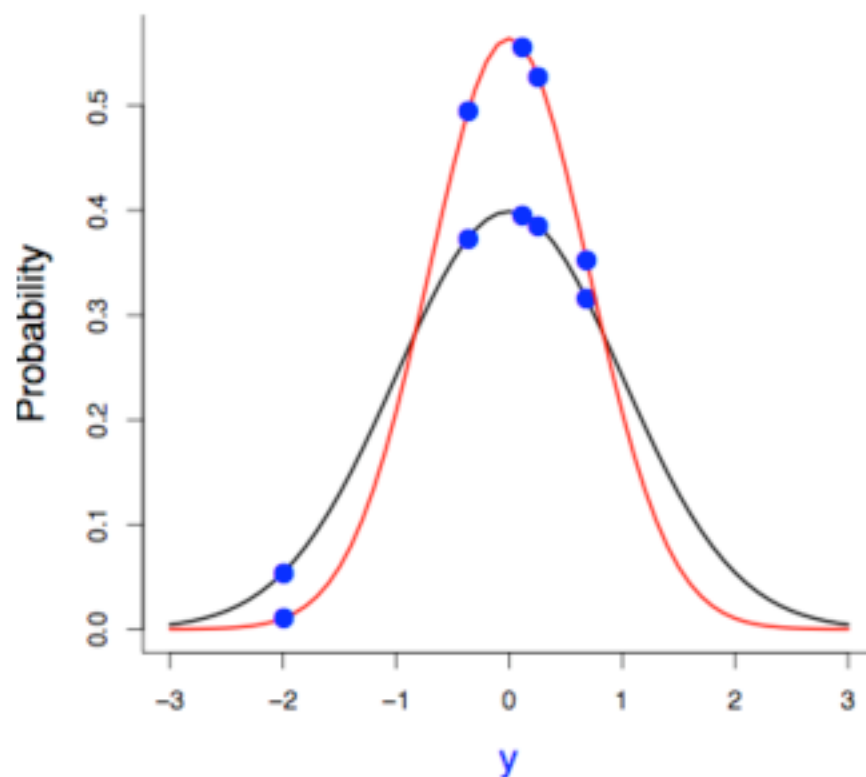
0.118

Likelihood

```
prod(dnorm(y, mean=0, sd=sqrt(0.5)))  
0.00054
```

μ

σ^2



0.256

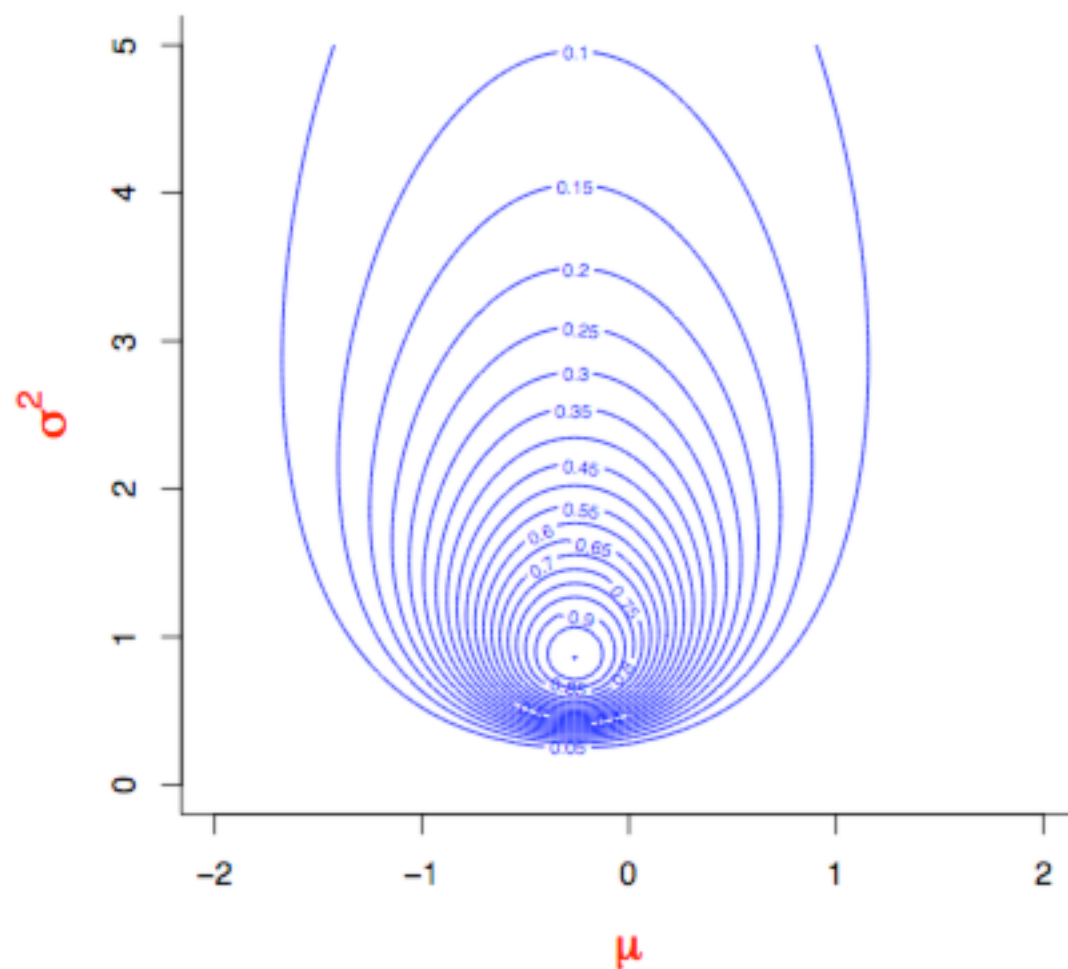
-1.995

-0.362

0.685

0.118

Likelihood



0.256

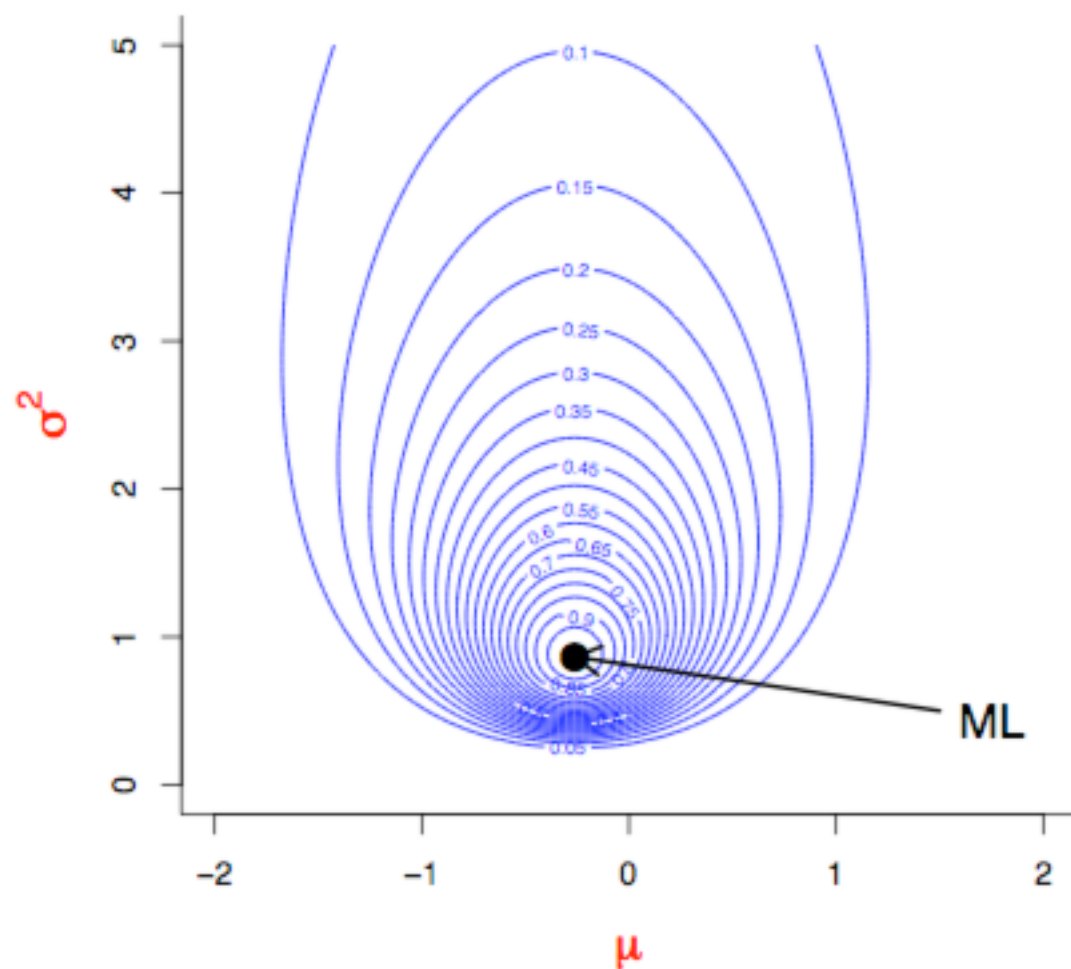
-1.995

-0.362

0.685

0.118

Maximum Likelihood



Maximum Likelihood

$$ML(\mu) = -0.2596 \quad ML(\sigma^2) = 0.8649$$

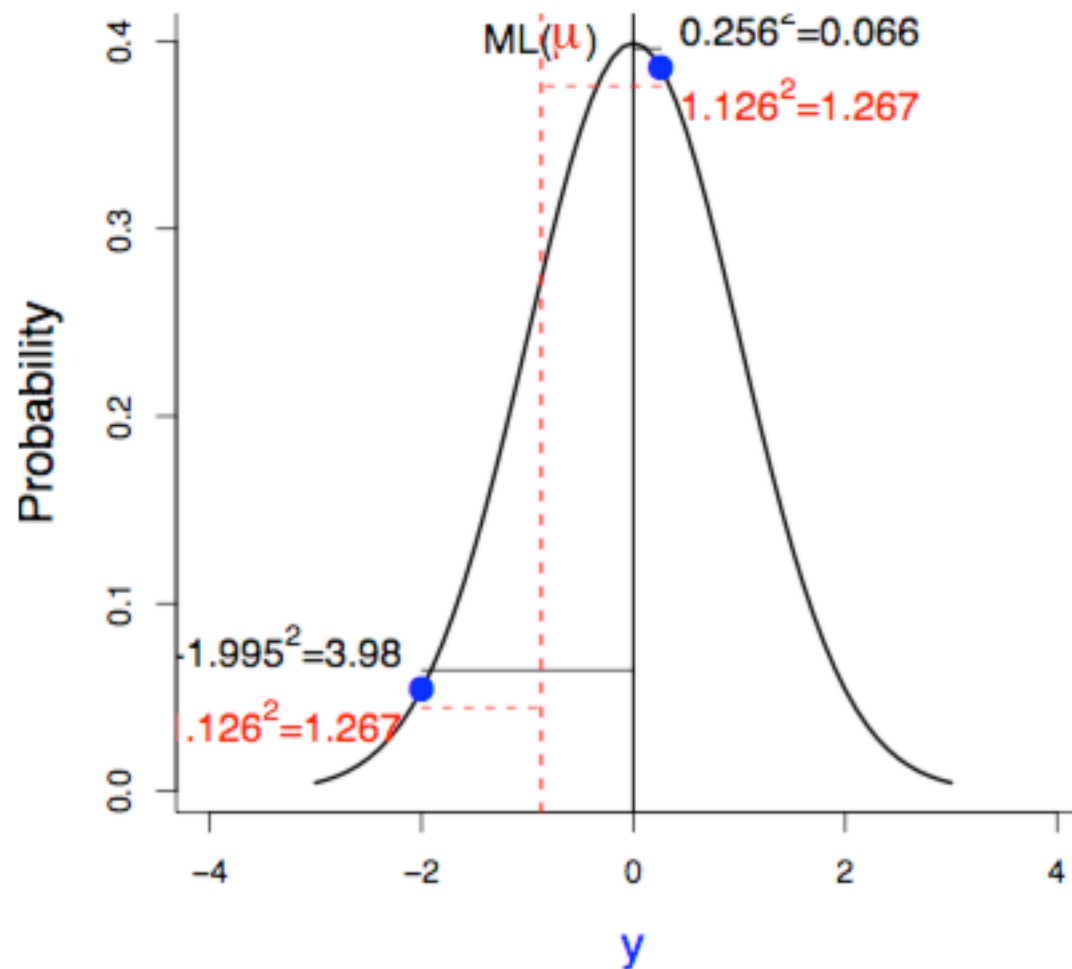
```
summary(glm(y~1))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.2596	0.4650	-0.558	0.606

(Dispersion parameter for gaussian 1.081121)

Restricted Maximum Likelihood



0.256

-1.995

-0.362

0.685

0.118

Bayesian

Pr(mean, variance | y) 0.256

Posterior distribution

μ

-1.995

σ^2

-0.362

0.685

0.118

Bayesian

$$\Pr(\text{mean, variance} \mid y) \quad 0.256$$

Posterior distribution

$$-1.995$$

=

$$\Pr(y \mid \text{mean, variance}) \quad -0.362$$

Likelihood

$$0.685$$

*

$$\Pr(\text{mean, variance}) \quad 0.118$$

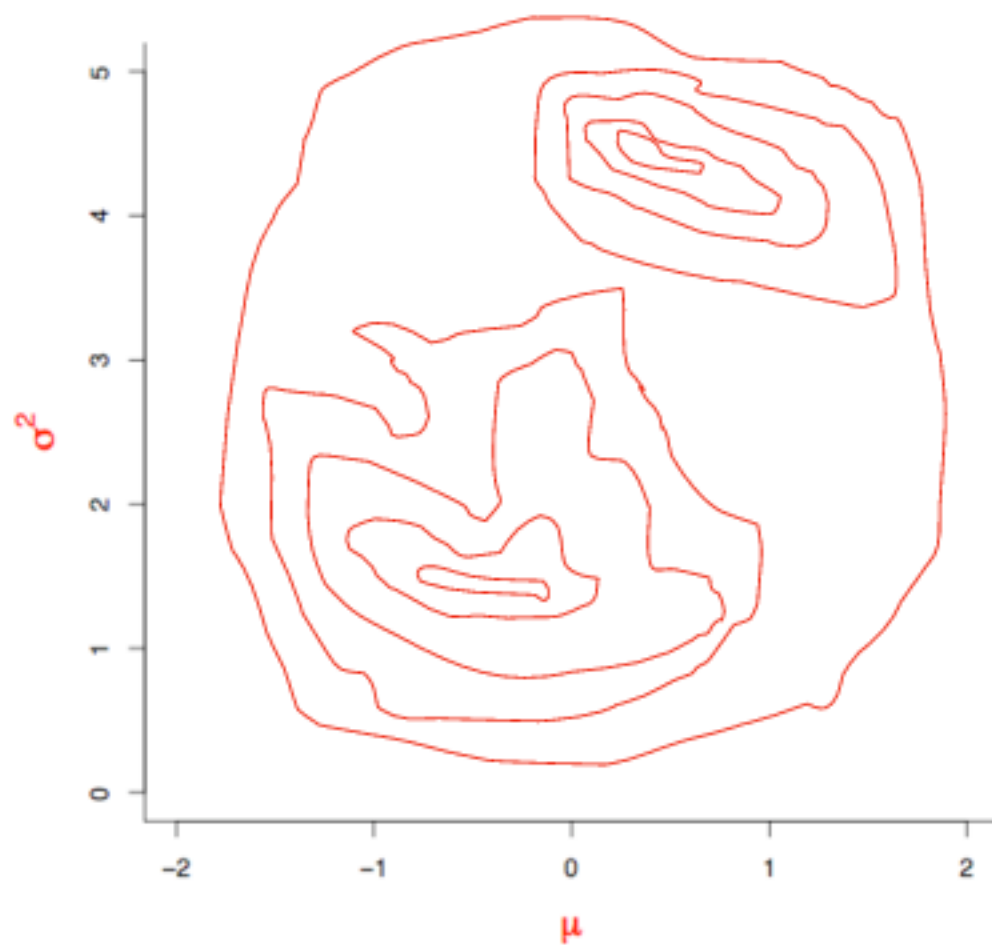
Prior

μ

σ^2

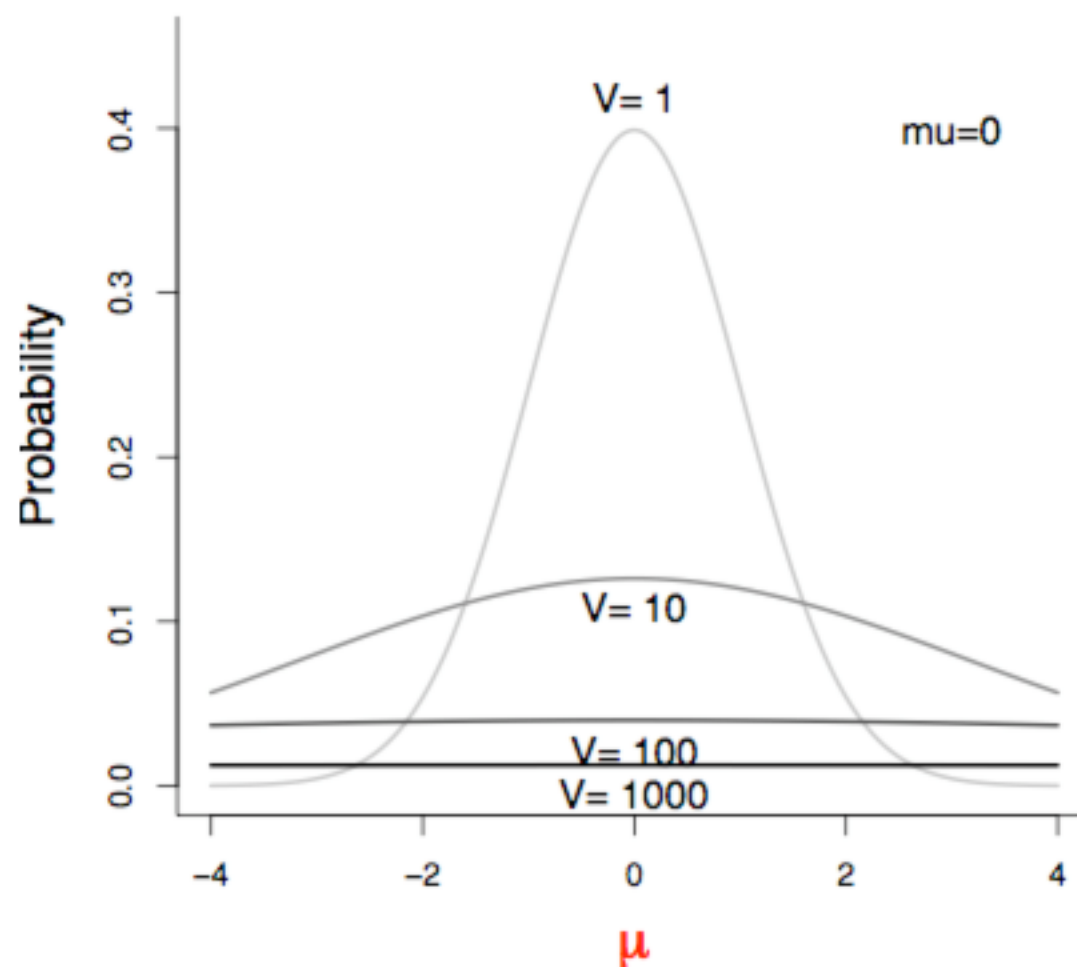
Prior

Pr(mean, variance)



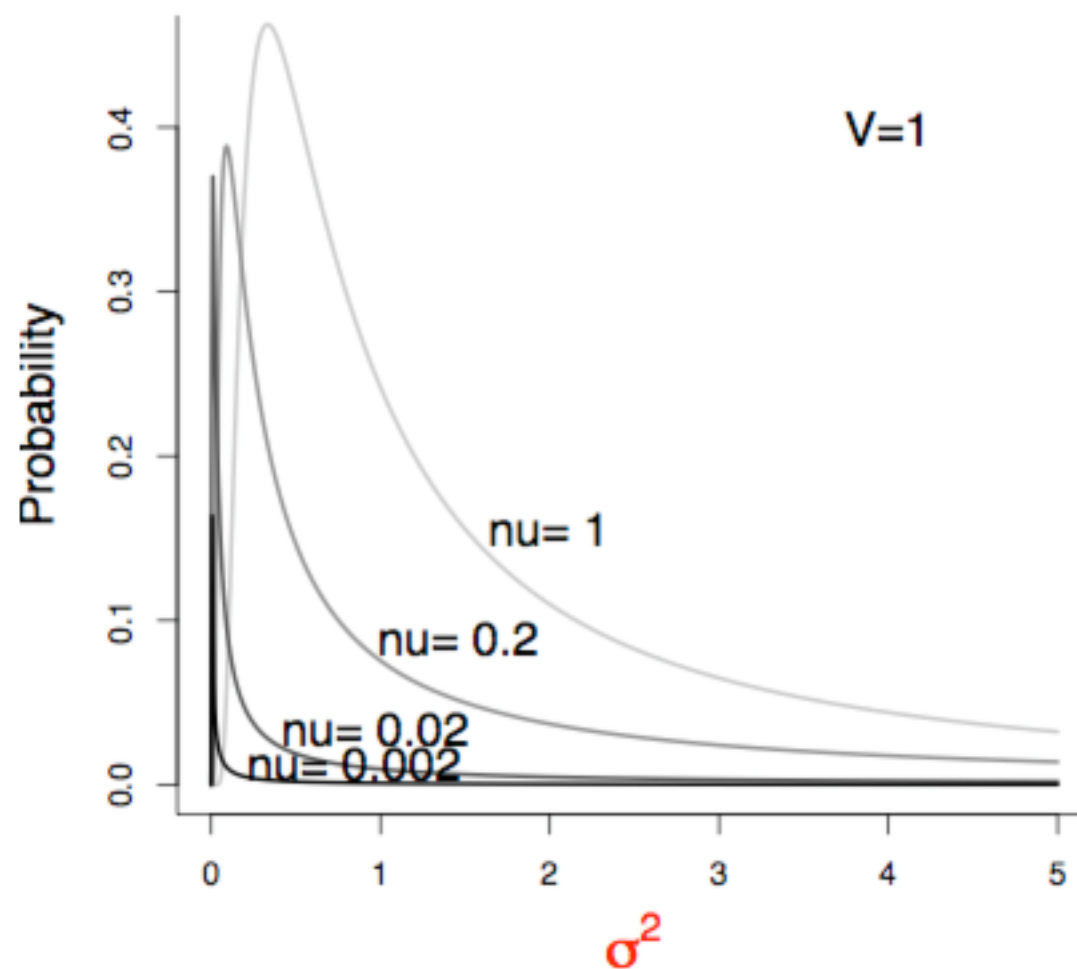
Prior

$$\text{Pr}(\text{mean}) \sim \text{Normal}(\text{B}\$\mu, \text{B}\$V)$$



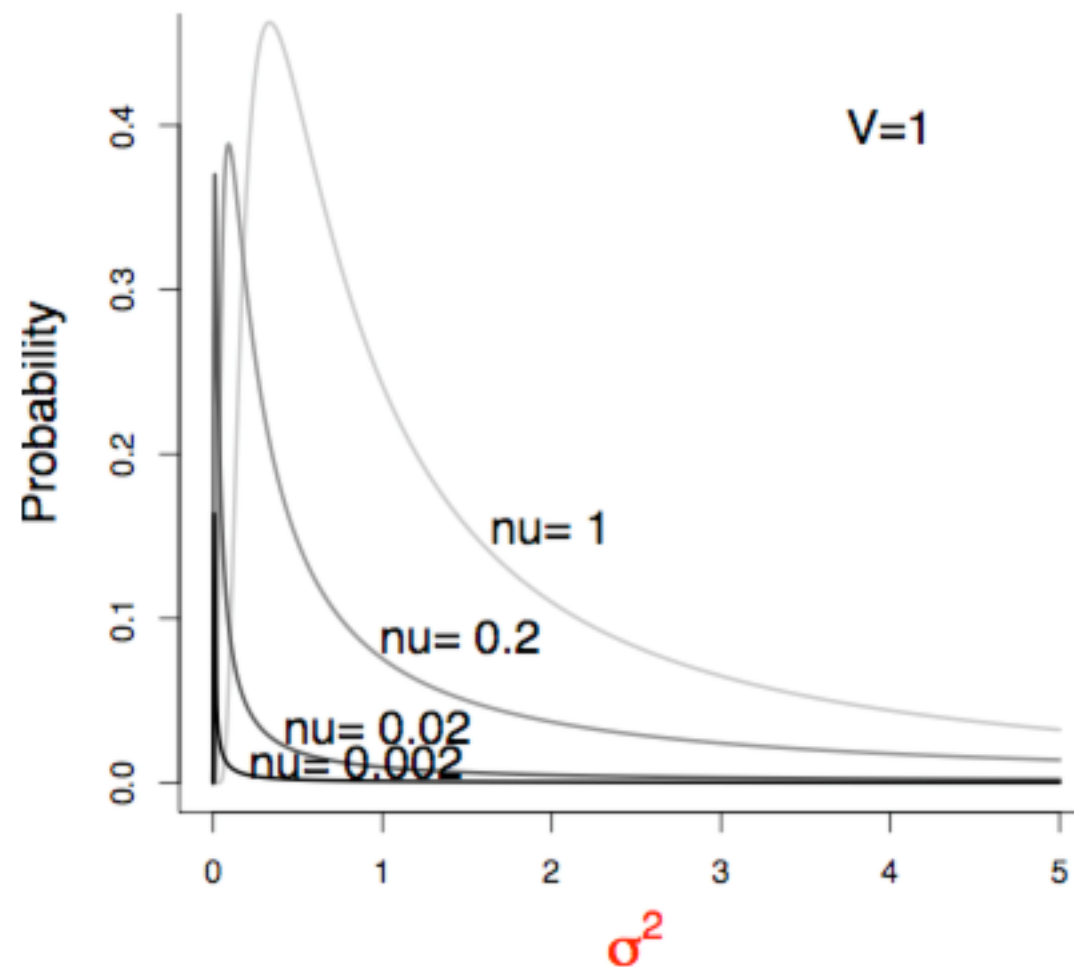
Prior

$\text{Pr}(\text{variance}) \sim \text{Inv-Wishart}(\text{R\$V}, \text{R\$nu})$



Prior

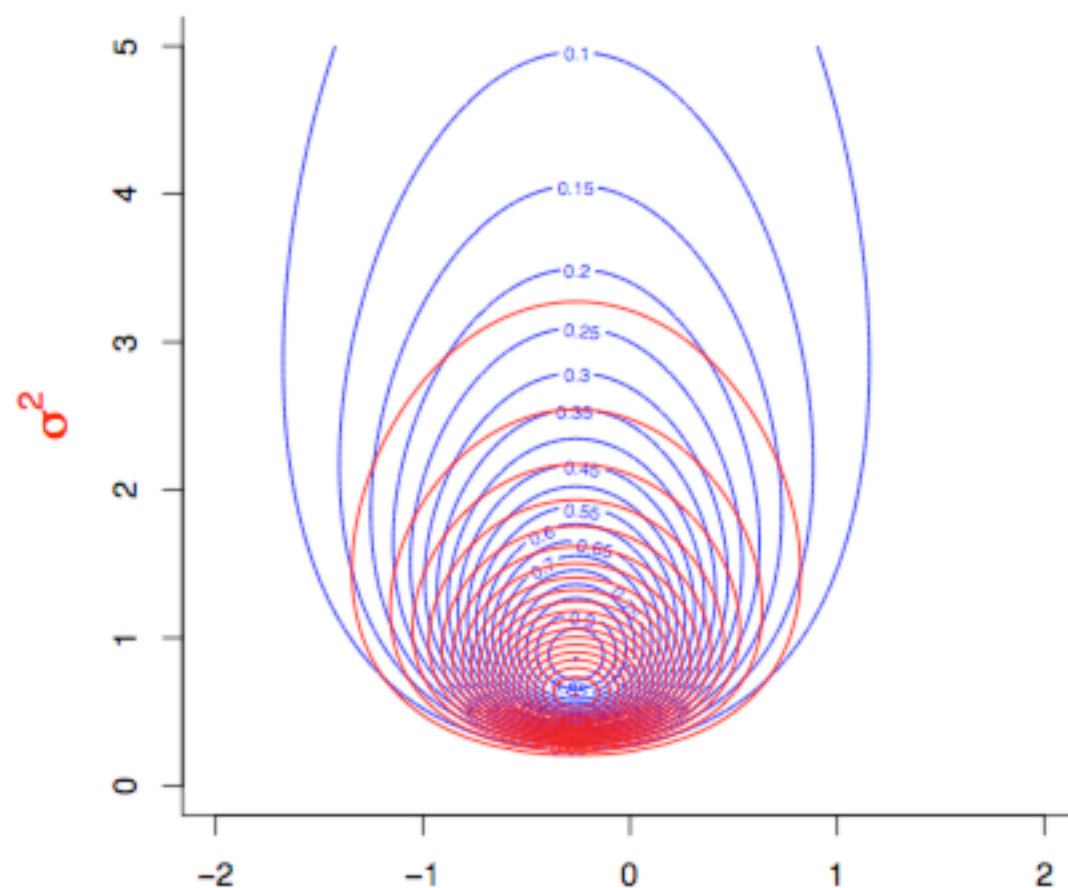
$\text{Pr}(\text{variance}) \sim \text{Inv-Wishart}(\text{R\$V}, \text{R\$nu})$



Posterior = Likelihood*Prior

Pr(**variance**) \sim IW(R\$V = 1, R\$nu = 0.002)

0.256



-1.995

-0.362

0.685

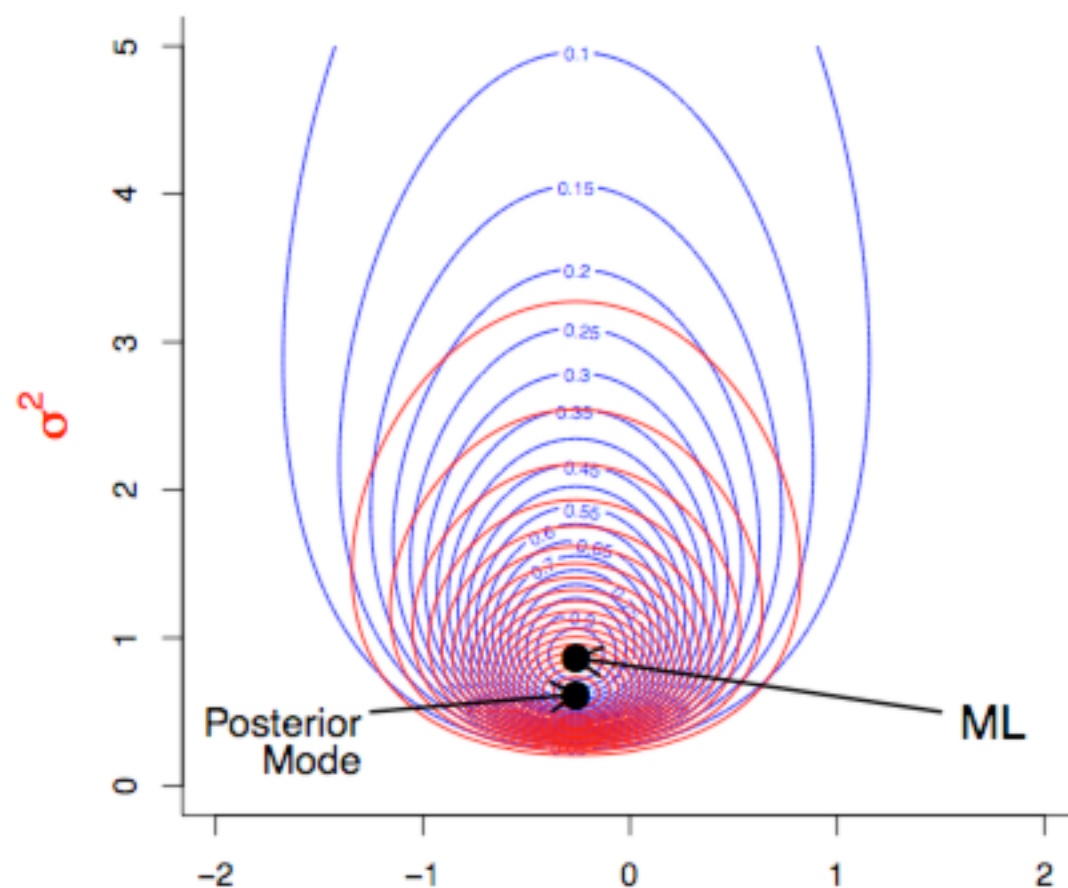
0.118

Pr(**mean**) \sim N(B\$mu = 0, B\$V = 10^8) μ

Posterior = Likelihood * Prior

$\Pr(\text{variance}) \sim \text{IW}(R\$V = 1, R\$nu = 0.002)$

0.256



-1.995

-0.362

0.685

0.118

$\Pr(\text{mean}) \sim N(B\$mu = 0, B\$V = 10^8)$

μ

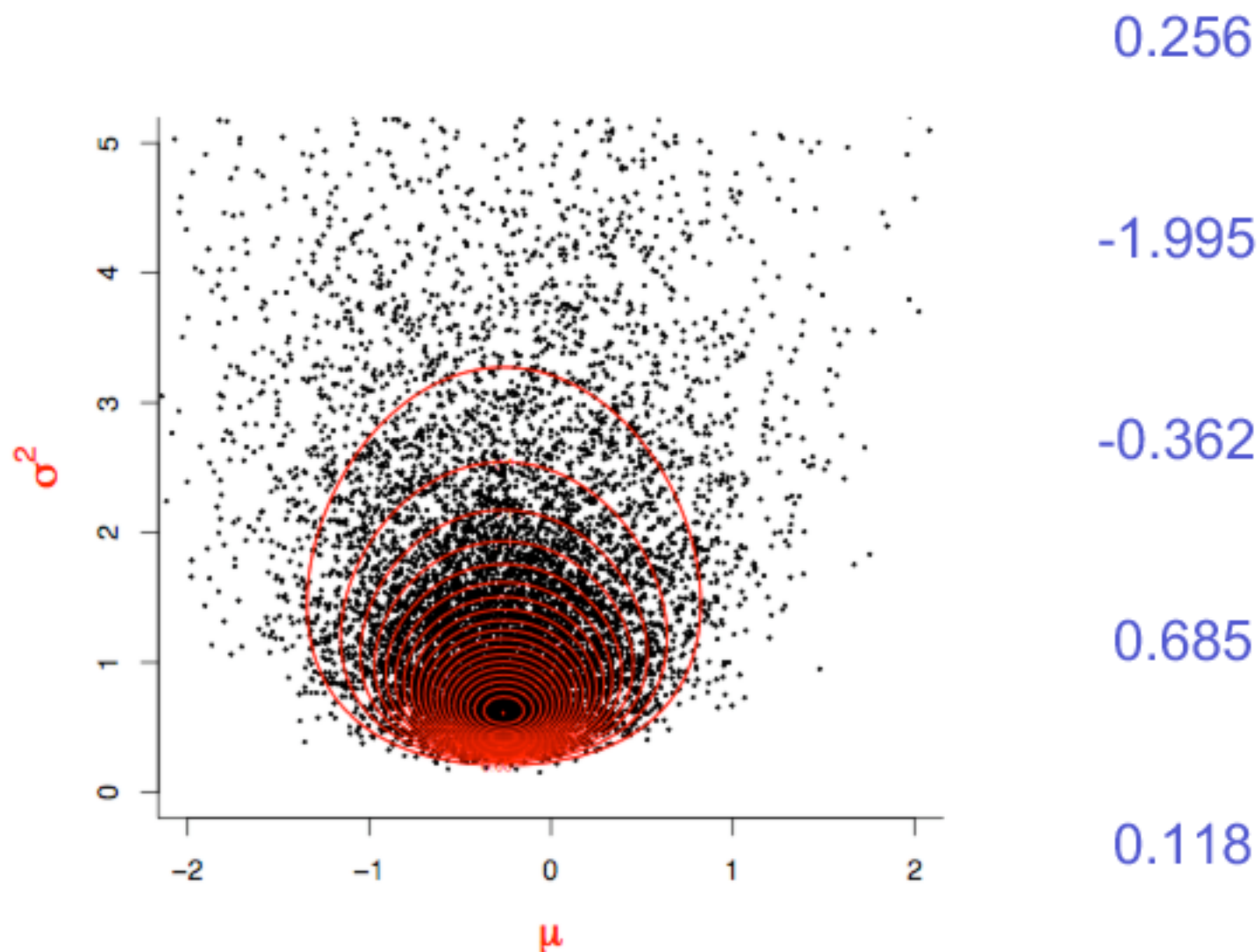
MCMC

```
prior=list(  
  B=list(mu=0, V=10^8)  
  R=list(V=1, nu=0.002)  
)
```

```
model1<-MCMCglmm(y~1, prior=prior)
```

```
points(model1$Sol, model1$VCV)
```

$$\text{Posterior} = \text{Likelihood} * \text{Prior}$$



Marginal Posterior Distribution

$\Pr(-2 < \mu < 2 \ \& \ \sigma^2 < 5 \mid y)$ 0.256

`table(-2 > model1$Sol < 2 & model1$VCV < 5)` -1.995

FALSE TRUE
0.0729 0.9271 -0.362

0.685

0.118

Marginal Posterior Distribution

$\Pr(-2 < \mu < 2 \ \& \ \sigma^2 < 5 \mid y)$ 0.256

`table(-2 > model1$Sol < 2 & model1$VCV < 5)` -1.995

FALSE TRUE
0.0729 0.9271 -0.362

$\Pr(\sigma^2 < 5 \mid y)$ 0.685

`table(model1$VCV < 5)`

FALSE TRUE
0.0693 0.9307 0.118

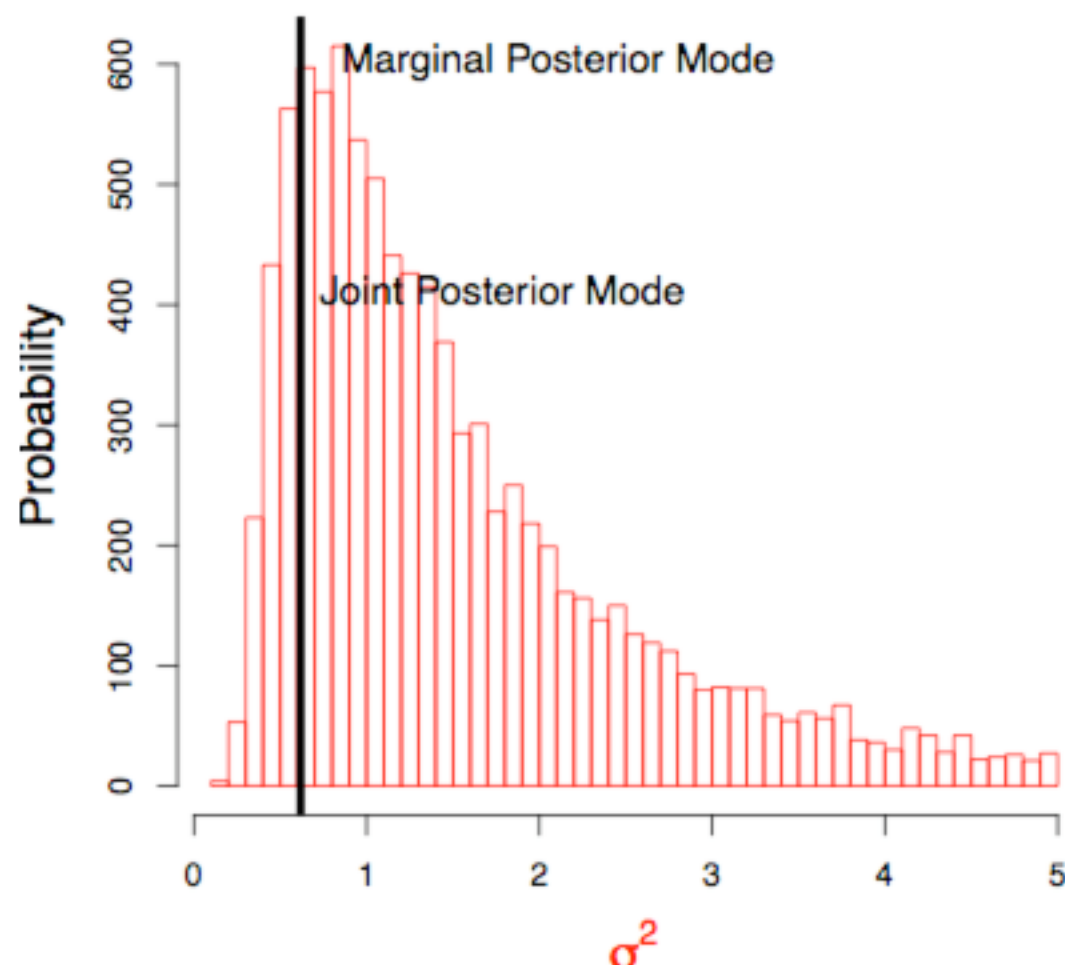
Marginal Posterior Distribution

$$\Pr(\sigma^2 \mid y) = \int \Pr(\mu, \sigma^2 \mid y) d\mu$$

Probability of the variance *given* the data, by
averaging over the uncertainty in the mean

Marginal Posterior Distribution

hist(model1\$VCV)



Improper Prior

A probability distribution must sum to one because a variable must have some value!

If we assume an equal probability for all values of the **mean**:

$$\text{Pr}(\text{mean}) \sim N(\mu = 0, V = \infty)$$

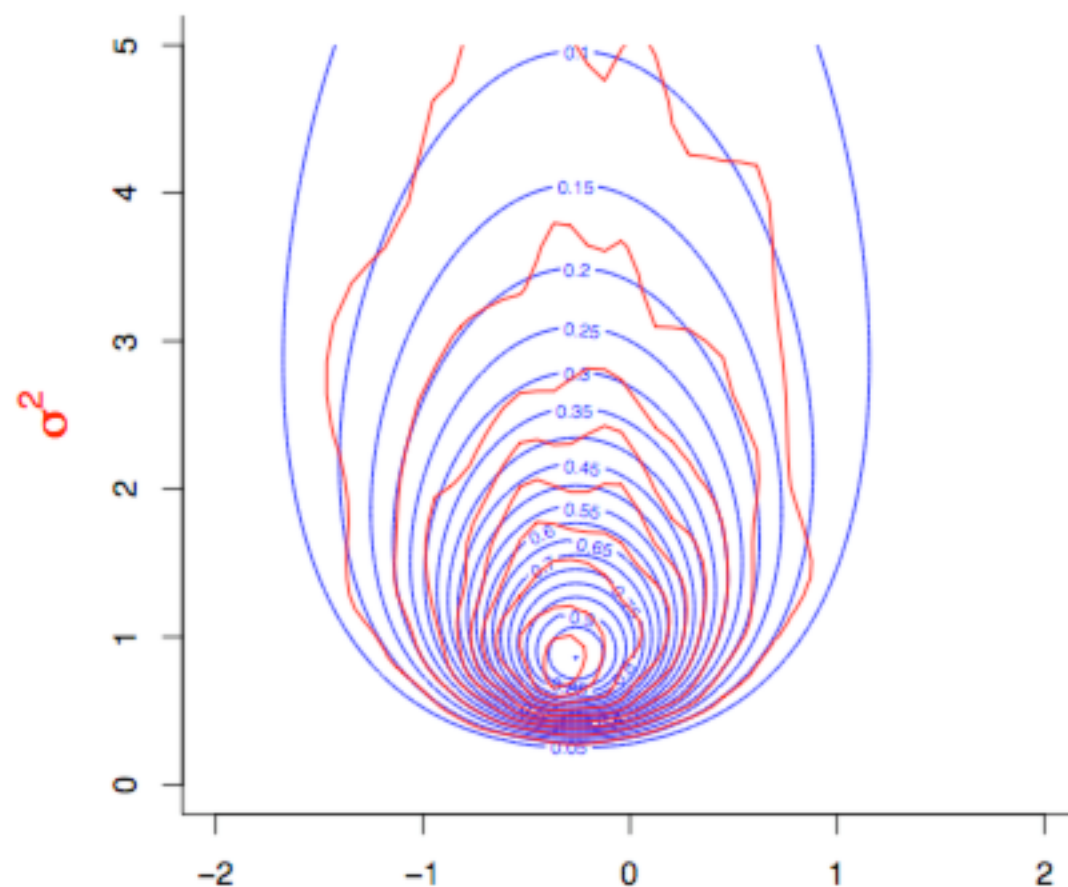
then this is not true, and the distribution is *improper*.

Improper priors can be useful but be very careful!

Flat Improper Prior

$\Pr(\text{variance}) \sim \text{IW}(R\$V = 0, R\$nu = 0)$

0.256



-1.995

-0.362

0.685

0.118

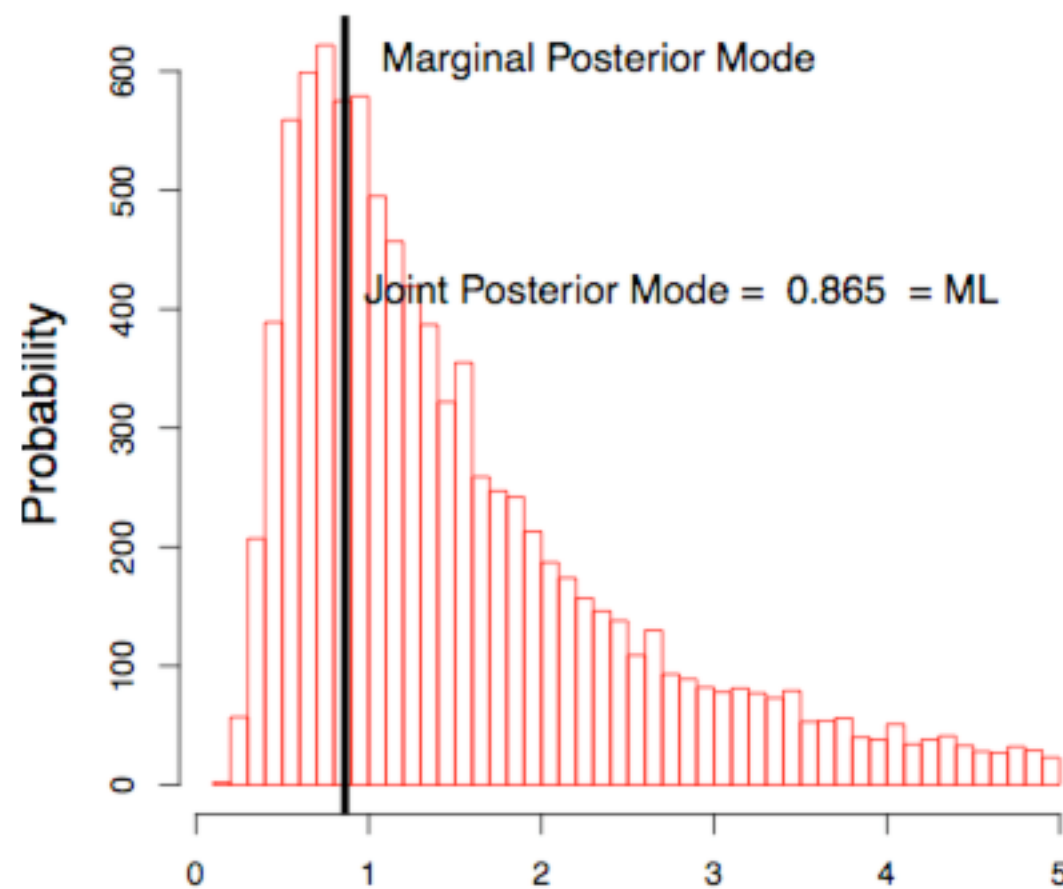
$\Pr(\text{mean}) \sim N(B\$mu = 0, B\$V = \infty)$

μ

Flat Improper Prior

$\Pr(\text{variance}) \sim \text{IW}(R\$V = 0, R\$nu = 0)$

0.256



-1.995

-0.362

0.685

0.118

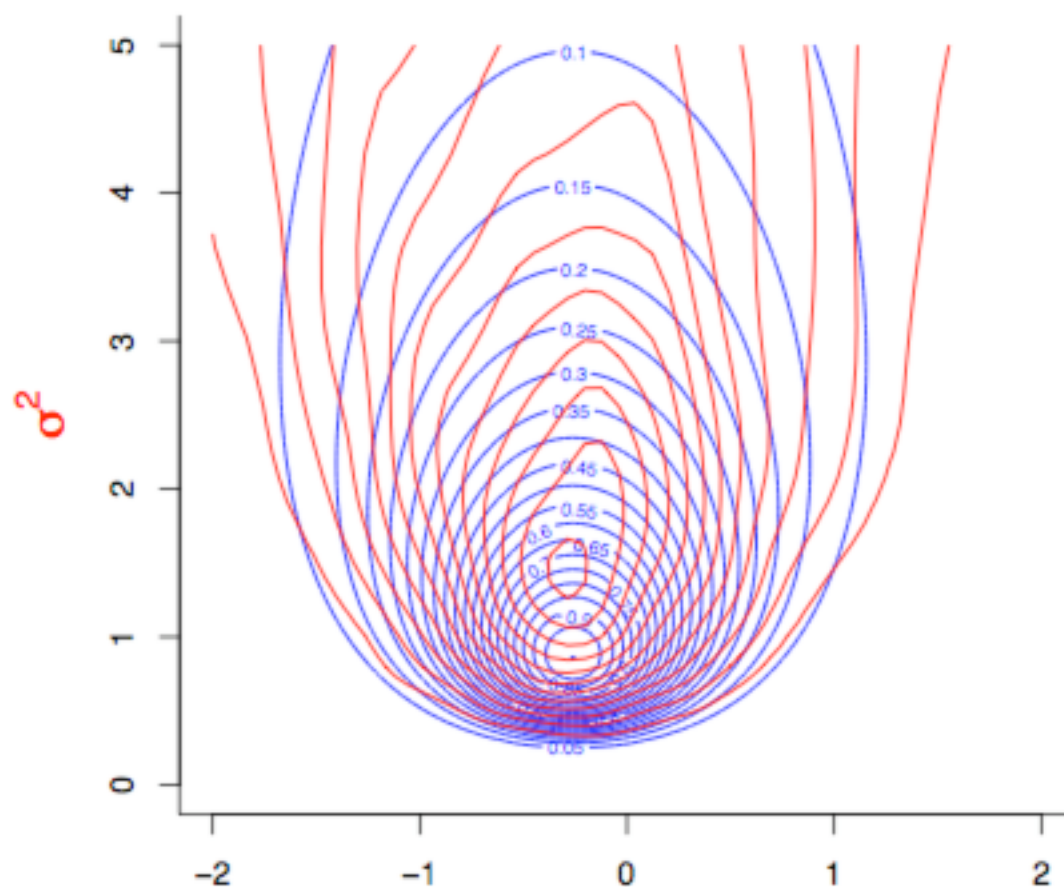
$\Pr(\text{mean}) \sim N(B\$mu = 0, B\$V = \infty)$

σ^2

Non-Informative Improper Prior

$\Pr(\text{variance}) \sim \text{IW}(R\$V = 0, R\$nu = -2)$

0.256



-1.995

-0.362

0.685

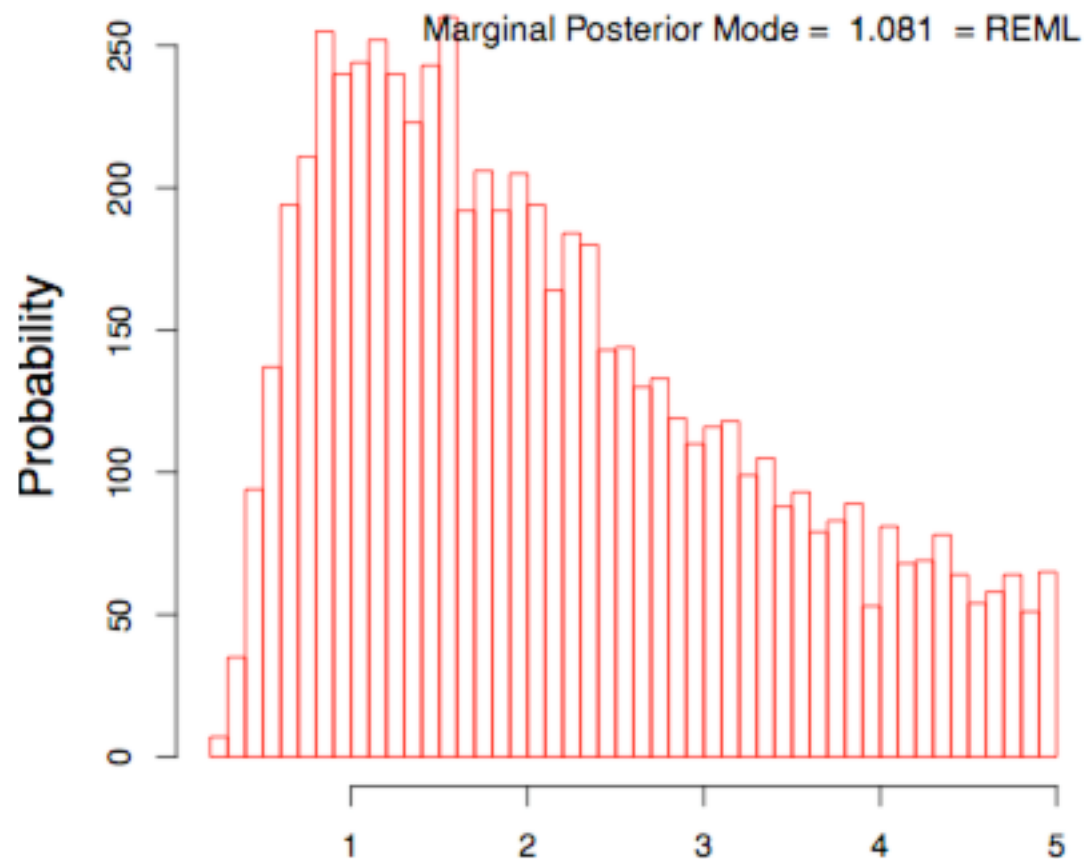
0.118

$\Pr(\text{mean}) \sim N(B\$mu = 0, B\$V = \infty)$

μ

Non-Informative Improper Prior

$\text{Pr}(\text{variance}) \sim \text{IW}(\text{R\$V} = 0, \text{R\$nu} = -2)$ 0.256



-1.995

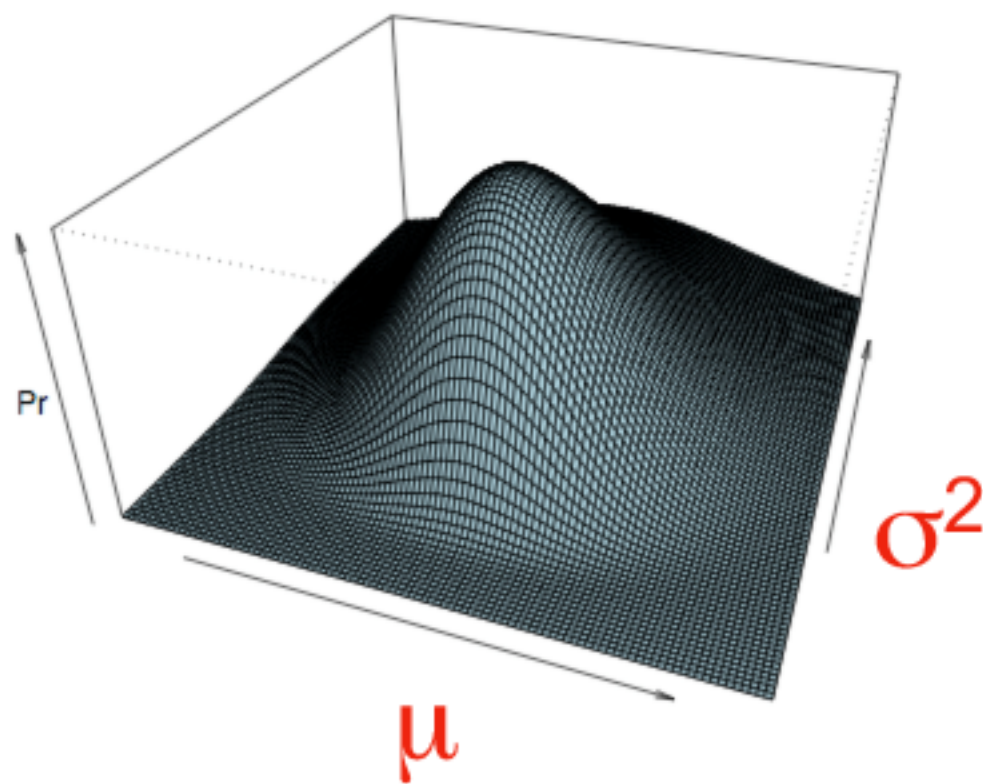
-0.362

0.685

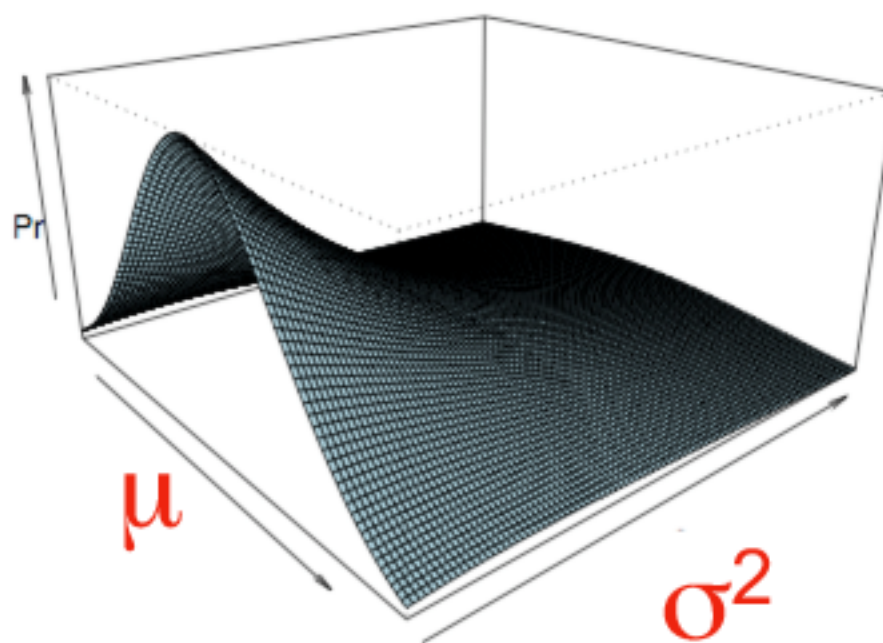
0.118

$\text{Pr}(\text{mean}) \sim \text{N}(\text{B\$mu} = 0, \text{B\$V} = \infty)$ σ^2

MCMC Diagnostics



MCMC Diagnostics



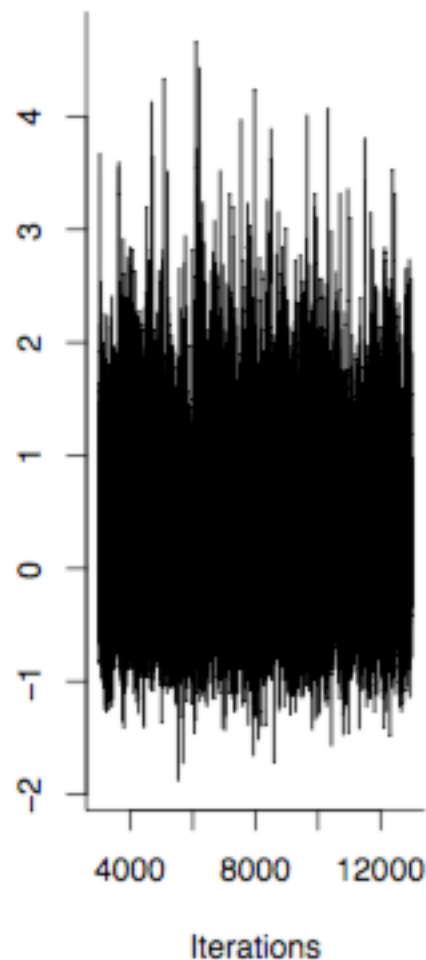
MCMC Diagnostics

```
plot(log(model1$VCV))
```

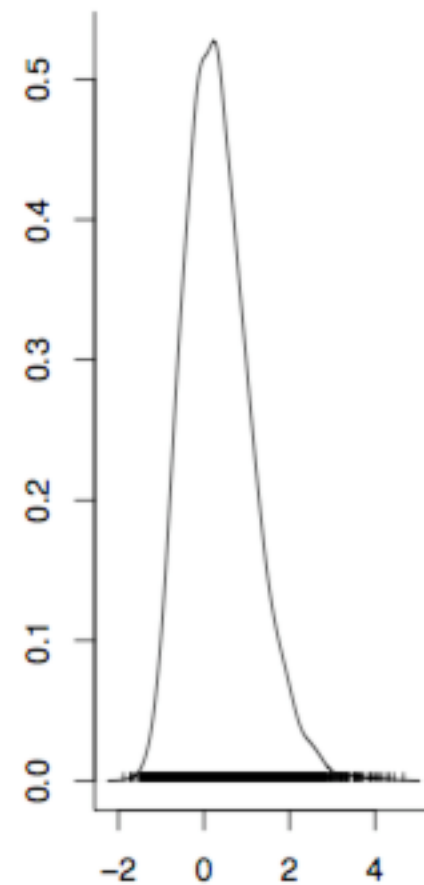
```
autocorr(model1$VCV)
```

```
      , ,units  
Lag 0 1.0000  
Lag 1 0.2047  
Lag 5 0.0011  
Lag 10 0.007  
Lag 50 0.013
```

Trace of units



Density of units



N = 10000 Bandwidth = 0.1287

MCMC Diagnostics

```
plot(model1$Sol)
```

```
autocorr(model1$Sol)
```

```
      , , (Intercept)
```

```
Lag 0 1.00000
```

```
Lag 1 0.01251
```

```
Lag 5 0.00736
```

```
Lag 10 0.0077
```

```
Lag 50 0.0060
```

