

RcppGSL: Easier **GSL** use from R via **Rcpp**

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Abstract

The GNU Scientific Library, or **GSL**, is a collection of numerical routines for scientific computing (Galassi et al., 2010). It is particularly useful for C and C++ programs as it provides a standard C interface to a wide range of mathematical routines such as special functions, permutations, combinations, fast fourier transforms, eigensystems, random numbers, quadrature, random distributions, quasi-random sequences, Monte Carlo integration, N-tuples, differential equations, simulated annealing, numerical differentiation, interpolation, series acceleration, Chebyshev approximations, root-finding, discrete Hankel transforms physical constants, basis splines and wavelets. There are over 1000 functions in total with an extensive test suite.

The **RcppGSL** package provides an easy-to-use interface between **GSL** data structures and R using concepts from **Rcpp** (Eddelbuettel and François, 2010) which is itself a package that eases the interfaces between R and C++.

1 Introduction

The GNU Scientific Library, or **GSL**, is a collection of numerical routines for scientific computing (Galassi et al., 2010). It is a rigourously developed and tested library providing support for a wide range of scientific or numerical tasks. Among the topics covered in the GSL are complex numbers, roots of polynomials, special functions, vector and matrix data structures, permutations, combinations, sorting, BLAS support, linear algebra, fast fourier transforms, eigensystems, random numbers, quadrature, random distributions, quasi-random sequences, Monte Carlo integration, N-tuples, differential equations, simulated annealing, numerical differentiation, interpolation, series acceleration, Chebyshev approximations, root-finding, discrete Hankel transforms least-squares fitting, minimization, physical constants, basis splines and wavelets.

Support for C programming with the GSL is readily available: the GSL itself is written in C and provides a C-language Application Programming Interface (API). Access from C++ is therefore possible, albeit not at the abstraction level that can be offered by dedicated C++ implementations.¹

The GSL is somewhat unique among numerical libraries. Its combination of broad coverage of scientific topics, serious implementation effort and the use of a FLOSS license have lead to a fairly wide usage of the library. As a concrete example, we can consider the the CRAN repository network for the R language and environment (R Development Core Team, 2010). CRAN contains over a dozen packages interfacing the GSL: **copula**, **dynamo**, **gsl**, **gstat**, **magnets**, **mvabund**, **QRMLib**, **RBrownie**, **RDieHarder**, **RHmm**, **segclust**, **surveillance**, and **topicmodels**. This is a clear indication that the GSL is popular among programmers using either the C or C++ language for solving problems applied science.

At the same time, the **Rcpp** package (Eddelbuettel and François, 2010) offers a higher-level abstraction between R and underlying C++ (or C) code. **Rcpp** permits R objects like vectors, matrices, lists, functions, environments, ..., to be manipulated directly at the C++ level, alleviates the needs for complicated and error-prone parameter passing and memory allocation. It also permits compact vectorised expressions similar to what can be written in R directly at the C++ level.

The **RcppGSL** package discussed here aims the help close the gap. It tries to offer access to GSL functions, in particular via the vector and matrix data structures used throughout the GSL, while staying closer to the ‘whole object model’ familiar to the R programmer.

The rest of paper is organised as follows. The next section shows a motivating example of a fast linear model fit routine using **GSL** functions. The following section discusses support for **GSL** vector types, which is followed by a section on matrices.

¹Several C++ wrappers for the GSL have been written over the years yet none reached a state of completion comparable to the GSL itself. Three such wrapping library are <http://cholm.home.cern.ch/cholm/misc/gslmm/>, <http://gslwrap.sourceforge.net/> and <http://code.google.com/p/gslcpp/>.

2 Motivation: FastLm

Fitting linear models is a key building block of analysing data and modeling. R has a very complete and feature-rich function in `lm()` which can provide a model fit as well as a number of diagnostic measures, either directly or via the corresponding `summary()` method for linear model fits. The `lm.fit()` function also provides a faster alternative (which is however recommended only for advanced users) which provides estimates only and fewer statistics for inference. This sometimes leads users to request a routine which is both fast and featureful enough. The `fastLm` routine shown here provides such an implementation. It uses the **GSL** for the least-squares fitting functions and therefore provides a nice example for **GSL** integration with R.

```
#include <RcppGSL.h>
#include <gsl/gsl_multifit.h>
#include <cmath>

extern "C" SEXP fastLm(SEXP ys, SEXP Xs) {

  try {

    RcppGSL::vector<double> y = ys;      // create gsl data structures from SEXP
    RcppGSL::matrix<double> X = Xs;

    int n = X.nrow(), k = X.ncol();
    double chisq;

    RcppGSL::vector<double> coef(k);      // to hold the coefficient vector
    RcppGSL::matrix<double> cov(k,k);     // and the covariance matrix

    // the actual fit requires working memory we allocate and free
    gsl_multifit_linear_workspace *work = gsl_multifit_linear_alloc (n, k);
    gsl_multifit_linear (X, y, coef, cov, &chisq, work);
    gsl_multifit_linear_free (work);

    // extract the diagonal as a vector view
    gsl_vector_view diag = gsl_matrix_diagonal(cov) ;

    // currently there is not a more direct interface in Rcpp::NumericVector
    // that takes advantage of wrap, so we have to do it in two steps
    Rcpp::NumericVector std_err ; std_err = diag;
    std::transform( std_err.begin(), std_err.end(), std_err.begin(), sqrt);

    Rcpp::List res = Rcpp::List::create(Rcpp::Named("coefficients") = coef,
                                         Rcpp::Named("stderr") = std_err,
                                         Rcpp::Named("df") = n - k);

    // free all the GSL vectors and matrices -- as these are really C data structures
    // we cannot take advantage of automatic memory management
    coef.free(); cov.free(); y.free(); X.free();

    return res;      // return the result list to R

  } catch( std::exception &ex ) {
    forward_exception_to_r( ex );
  } catch(...) {
    ::Rf_error( "c++ exception (unknown reason)" );
  }
  return R_NilValue; // -Wall
}
```

We first initialize a `RcppGSL` vector and matrix, each templated to the standard numeric type `double` (and the **GSL** supports other types ranging from lower precision floating point to signed and unsigned integers

as well as complex numbers). We then reserve another vector and matrix to hold the resulting coefficient estimates as well as the estimate of the covariance matrix. Next, we allocate workspace using a GSL routine, fit the linear model and free the workspace. The next step involves extracting the diagonal element from the covariance matrix. We then employ a so-called iterator—a common C++ idiom from the Standard Template Library (STL)—to iterate over the vector of diagonal and transforming it by applying the square root function to compute our standard error of the estimate. Finally we create a named list with the return value before we free temporary memory allocation (a step that has to be done because the underlying objects are really C objects conforming to the **GSL** interface and hence without the automatic memory management we could have with C++ vector or matrix structures as used through the **Rcpp** package) and return the result to R.

We should note that **RcppArmadillo** (François, Eddelbuettel, and Bates, 2010) implements a matching `fastLm` function using the Armadillo library by Sanderson (2010), and can do so with more compact code due to C++ features.

3 Vectors

This section details the different vector representations, starting with their definition inside the **GSL**. We then discuss our layering before showing how the two types map. A discussion of read-only ‘vector view’ classes concludes the section.

3.1 GSL Vectors

GSL defines various vector types to manipulate one-dimensional data, similar to R arrays. For example the `gsl_vector` and `gsl_vector_int` structs are defined as:

```
typedef struct{
    size_t size;
    size_t stride;
    double * data;
    gsl_block * block;
    int owner;
} gsl_vector;

typedef struct {
    size_t size;
    size_t stride;
    int * data;
    gsl_block_int * block;
    int owner;
}
gsl_vector_int;
```

A typical use of the `gsl_vector` struct is given below:

```
int i;
gsl_vector * v = gsl_vector_alloc (3); // allocate a gsl_vector of size 3

for (i = 0; i < 3; i++) { // fill the vector
    gsl_vector_set (v, i, 1.23 + i);
}

double sum = 0.0 ; // access elements
for (i = 0; i < 3; i++) {
    sum += gsl_vector_get( v, i ) ;
}

gsl_vector_free (v); // free the memory
```

3.2 RcppGSL::vector

RcppGSL defines the template `RcppGSL::vector<T>` to manipulate `gsl_vector` pointers taking advantage of C++ templates. Using this template type, the previous example now becomes:

```
int i;
RcppGSL::vector<double> v(3);           // allocate a gsl_vector of size 3

for (i = 0; i < 3; i++) {               // fill the vector
    v[i] = 1.23 + i ;
}

double sum = 0.0 ;                       // access elements
for (i = 0; i < 3; i++) {
    sum += v[i] ;
}

v.free() ;                               // free the memory
```

The class `RcppGSL::vector<double>` is a smart pointer, that can be used anywhere where a raw pointer `gsl_vector` can be used, such as the `gsl_vector_set` and `gsl_vector_get` functions above.

Beyond the convenience of a nicer syntax for allocation and release of memory, the `RcppGSL::vector` template facilitates interchange of **GSL** vectors with **Rcpp** objects, and hence **R** objects. The following example defines a `.Call` compatible function called `sum_gsl_vector_int` that operates on a `gsl_vector_int` through the `RcppGSL::vector<int>` template specialization:

```
RCPP_FUNCTION_1( int, sum_gsl_vector_int, RcppGSL::vector<int> vec){
    int res = std::accumulate( vec.begin(), vec.end(), 0 ) ;
    vec.free() ; // we need to free vec after use
    return res ;
}
```

The function can then simply be called from R :

```
> .Call( "sum_gsl_vector_int", 1:10 )
[1] 55
```

A second example shows a simple function that grabs elements of an R list as `gsl_vector` objects using implicit conversion mechanisms of **Rcpp**

```
RCPP_FUNCTION_1( double, gsl_vector_sum_2, Rcpp::List data ){
    // grab "x" as a gsl_vector through the RcppGSL::vector<double> class
    RcppGSL::vector<double> x = data["x"] ;

    // grab "y" as a gsl_vector through the RcppGSL::vector<int> class
    RcppGSL::vector<int> y = data["y"] ;
    double res = 0.0 ;
    for( size_t i=0; i< x->size; i++){
        res += x[i] * y[i] ;
    }

    // as usual with GSL, we need to explicitly free the memory
    x.free() ;
    y.free() ;

    // return the result
    return res ;
}
```

called from R :

```
> data <- list( x = seq(0,1,length=10), y = 1:10 )
> .Call( "gsl_vector_sum_2", data )
[1] 36.66667
```

3.3 Mapping

Table 1 shows the mapping between types defined by the **GSL** and their corresponding types in the **RcppGSL** package.

gsl vector	RcppGSL
<code>gsl_vector</code>	<code>RcppGSL::vector<double></code>
<code>gsl_vector_int</code>	<code>RcppGSL::vector<int></code>
<code>gsl_vector_float</code>	<code>RcppGSL::vector<float></code>
<code>gsl_vector_long</code>	<code>RcppGSL::vector<long></code>
<code>gsl_vector_char</code>	<code>RcppGSL::vector<char></code>
<code>gsl_vector_complex</code>	<code>RcppGSL::vector<gsl_complex></code>
<code>gsl_vector_complex_float</code>	<code>RcppGSL::vector<gsl_complex_float></code>
<code>gsl_vector_complex_long_double</code>	<code>RcppGSL::vector<gsl_complex_long_double></code>
<code>gsl_vector_long_double</code>	<code>RcppGSL::vector<long double></code>
<code>gsl_vector_short</code>	<code>RcppGSL::vector<short></code>
<code>gsl_vector_uchar</code>	<code>RcppGSL::vector<unsigned char></code>
<code>gsl_vector_uint</code>	<code>RcppGSL::vector<unsigned int></code>
<code>gsl_vector_ushort</code>	<code>RcppGSL::vector<unsigned short></code>
<code>gsl_vector_ulong</code>	<code>RcppGSL::vector<unsigned long></code>

Table 1: Correspondance between **GSL** vector types and templates defined in **RcppGSL**.

3.4 Vector Views

Several **GSL** algorithms return **GSL** vector views as their result type. **RcppGSL** defines the template class `RcppGSL::vector_view` to handle vector views using C++ syntax.

```
extern "C" SEXP test_gsl_vector_view(){
  int n = 10 ;
  RcppGSL::vector<double> v(n) ;
  for( int i=0 ; i<n; i++){
    v[i] = i ;
  }
  RcppGSL::vector_view<double> v_even = gsl_vector_subvector_with_stride(v,0,2,n/2);
  RcppGSL::vector_view<double> v_odd = gsl_vector_subvector_with_stride(v,1,2,n/2);

  List res = List::create(
    _["even"] = v_even,
    _["odd"] = v_odd
  ) ;
  v.free() ; // we only need to free v, the views do not own data
  return res ;
}
```

As with vectors, C++ objects of type `RcppGSL::vector_view` can be converted implicitly to their associated **GSL** view type. Table 2 displays the pairwise correspondance so that the C++ objects can be passed to compatible **GSL** algorithms.

The vector view class also contains a conversion operator to automatically transform the data of the view object to a **GSL** vector object. This enables use of vector views where **GSL** would expect a vector.

gsl vector views	RcppGSL
<code>gsl_vector_view</code>	<code>RcppGSL::vector_view<double></code>
<code>gsl_vector_view_int</code>	<code>RcppGSL::vector_view<int></code>
<code>gsl_vector_view_float</code>	<code>RcppGSL::vector_view<float></code>
<code>gsl_vector_view_long</code>	<code>RcppGSL::vector_view<long></code>
<code>gsl_vector_view_char</code>	<code>RcppGSL::vector_view<char></code>
<code>gsl_vector_view_complex</code>	<code>RcppGSL::vector_view<gsl_complex></code>
<code>gsl_vector_view_complex_float</code>	<code>RcppGSL::vector_view<gsl_complex_float></code>
<code>gsl_vector_view_complex_long_double</code>	<code>RcppGSL::vector_view<gsl_complex_long_double></code>
<code>gsl_vector_view_long_double</code>	<code>RcppGSL::vector_view<long double></code>
<code>gsl_vector_view_short</code>	<code>RcppGSL::vector_view<short></code>
<code>gsl_vector_view_uchar</code>	<code>RcppGSL::vector_view<unsigned char></code>
<code>gsl_vector_view_uint</code>	<code>RcppGSL::vector_view<unsigned int></code>
<code>gsl_vector_view_ushort</code>	<code>RcppGSL::vector_view<unsigned short></code>
<code>gsl_vector_view_ulong</code>	<code>RcppGSL::vector_view<unsigned long></code>

Table 2: Correspondance between **GSL** vector view types and templates defined in **RcppGSL**.

4 Matrices

The **GSL** also defines a set of matrix data types : `gsl_matrix`, `gsl_matrix_int` etc ... for which **RcppGSL** defines a corresponding convenience C++ wrapper generated by the `RcppGSL::matrix` template.

4.1 Creating matrices

The `RcppGSL::matrix` template exposes three constructors.

```
// convert an R matrix to a GSL matrix
matrix( SEXP x) throw(::Rcpp::not_compatible)

// encapsulate a GSL matrix pointer
matrix( gsl_matrix* x)

// create a new matrix with the given number of rows and columns
matrix( int nrow, int ncol)
```

4.2 Implicit conversion

`RcppGSL::matrix` defines implicit conversion to a pointer to the associated **GSL** matrix type, as well as dereferencing operators, making the class `RcppGSL::matrix` look and feel like a pointer to a **GSL** matrix type.

```
gsltype* data ;
operator gsltype*(){ return data ; }
gsltype* operator->() const { return data; }
gsltype& operator*() const { return *data; }
```

4.3 Indexing

Indexing of **GSL** matrices is usually the task of the functions `gsl_matrix_get`, `gsl_matrix_int_get`, ... and `gsl_matrix_set`, `gsl_matrix_int_set`, ...

RcppGSL takes advantage of both operator overloading and templates to make indexing a **GSL** matrix much more convenient.

```
RcppGSL::matrix<int> mat(10,10);           // create a matrix of size 10x10

for( int i=0; i<10: i++) {                 // fill the diagonal
    mat(i,i) = i ;
}
```

4.4 Methods

The `RcppGSL::matrix` type also defines the following member functions:

- `nrow` extracts the number of rows
- `ncol` extract the number of columns
- `size` extracts the number of elements
- `free` releases the memory

4.5 Matrix views

Similar to the vector views discussed above, the **RcppGSL** also provides an implicit conversion operator which returns the underlying matrix stored in the matrix view class.

5 Using RcppGSL in your package

The **RcppGSL** package contains a complete example providing a single function `colNorm` which computes a norm for each column of a supplied matrix. This example adapts a matrix example from the GSL manual that has been chose merely as a means to showing how to set up a package to use **RcppGSL**.

Needless to say, we could compute such a matrix norm easily in R using existing facilities. One such possibility is a simple `apply(M, 2, function(x) sqrt(sum(x^2)))` as shown on the corresponding help page in the example package inside **RcppGSL**. One point in favour of using the **GSL** code is that it employs a BLAS function so on sufficiently large matrices, and with suitable BLAS libraries installed, this variant could be faster due to the optimised code in high-performance BLAS libraries and/or the inherent parallelism a multi-core BLAS variant which compute compute the vector norm in parallel. On all ‘reasonable’ matrix sizes, however, the performance difference should be negligible.

5.1 The configure script

5.1.1 Using autoconf

Using **RcppGSL** means employing both the **GSL** and R. We may need to find the location of the **GSL** headers and library, and this done easily from a `configure` source script which `autoconf` generates from a `configure.in` source file such as the following:

```

AC_INIT([RcppGSLExample], 0.1.0)

## Use gsl-config to find arguments for compiler and linker flags
##
## Check for non-standard programs: gsl-config(1)
AC_PATH_PROG([GSL_CONFIG], [gsl-config])
## If gsl-config was found, let's use it
if test "${GSL_CONFIG}" != ""; then
    # Use gsl-config for header and linker arguments (without BLAS which we get from R)
    GSL_CFLAGS='${GSL_CONFIG} --cflags'
    GSL_LIBS='${GSL_CONFIG} --libs-without-cblas'
else
    AC_MSG_ERROR([gsl-config not found, is GSL installed?])
fi

## Use Rscript to query Rcpp for compiler and linker flags
## link flag providing library as well as path to library, and optionally rpath
RCPP_LDFLAGS='${R_HOME}/bin/Rscript -e 'Rcpp::LdFlags()' '

# Now substitute these variables in src/Makevars.in to create src/Makevars
AC_SUBST(GSL_CFLAGS)
AC_SUBST(GSL_LIBS)
AC_SUBST(RCPP_LDFLAGS)

AC_OUTPUT(src/Makevars)

```

Such a source `configure.in` gets converted into a script `configure` by invoking the `autoconf` program.

5.1.2 Using functions provided by RcppGSL

RcppGSL provides R functions that allows one to retrieve the same information. Therefore the `configure` script can also be written as:

```

#!/bin/sh

GSL_CFLAGS='${R_HOME}/bin/Rscript -e "RcppGSL::CFlags()"'
GSL_LIBS='${R_HOME}/bin/Rscript -e "RcppGSL::LdFlags()"'
RCPP_LDFLAGS='${R_HOME}/bin/Rscript -e "Rcpp::LdFlags()"'

sed -e "s|@GSL_LIBS@|${GSL_LIBS}|" \
    -e "s|@GSL_CFLAGS@|${GSL_CFLAGS}|" \
    -e "s|@RCPP_LDFLAGS@|${RCPP_LDFLAGS}|" \
    src/Makevars.in > src/Makevars

```

Similarly, the `configure.win` for windows can be written as:

```

GSL_CFLAGS='${R_HOME}/bin${R_ARCH_BIN}/Rscript.exe -e "RcppGSL::CFlags()"'
GSL_LIBS='${R_HOME}/bin${R_ARCH_BIN}/Rscript.exe -e "RcppGSL::LdFlags()"'
RCPP_LDFLAGS='${R_HOME}/bin${R_ARCH_BIN}/Rscript.exe -e "Rcpp::LdFlags()"'

sed -e "s|@GSL_LIBS@|${GSL_LIBS}|" \
    -e "s|@GSL_CFLAGS@|${GSL_CFLAGS}|" \
    -e "s|@RCPP_LDFLAGS@|${RCPP_LDFLAGS}|" \
    src/Makevars.in > src/Makevars.win

```


5.2 The src directory

The C++ source file takes the matrix supplied from R and applies the **GSL** function to each column.

```
#include <RcppGSL.h>
#include <gsl/gsl_matrix.h>
#include <gsl/gsl_blas.h>

extern "C" SEXP colNorm(SEXP sM) {

  try {

    RcppGSL::matrix<double> M = sM;      // create gsl data structures from SEXP
    int k = M.ncol();
    Rcpp::NumericVector n(k);            // to store results

    for (int j = 0; j < k; j++) {
      RcppGSL::vector_view<double> colview = gsl_matrix_column (M, j);
      n[j] = gsl_blas_dnorm2(colview);
    }
    M.free();
    return n;                            // return vector

  } catch( std::exception &ex ) {
    forward_exception_to_r( ex );

  } catch(...) {
    ::Rf_error( "c++ exception (unknown reason)" );
  }

  return R_NilValue; // -Wall
}
```

The Makevars.in file governs the compilation and uses the values supplied by **configure** during build-time:

```
# set by configure
GSL_CFLAGS = @GSL_CFLAGS@
GSL_LIBS   = @GSL_LIBS@
RCPP_LDFLAGS = @RCPP_LDFLAGS@

# combine with standard arguments for R
PKG_CPPFLAGS = $(GSL_CFLAGS)
PKG_LIBS = $(GSL_LIBS) $(RCPP_LDFLAGS)
```

The variables surrounded by `will` be filled by **configure** during package build-time.

5.3 The R directory

The R source is very simply: a single matrix is passed to C++:

```
> colNorm <- function(M) {
+   stopifnot(is.matrix(M))
+   res <- .Call("colNorm", M, package="RcppGSLExample")
+ }
```

6 Using RcppGSL with inline

The **inline** package (Sklyar, Murdoch, Smith, Eddelbuettel, and François, 2010) is very helpful for prototyping code in C, C++ or Fortran as it takes care of code compilation, linking and dynamic loading directly from R.

It is being used extensively by **Rcpp**, for example in the numerous unit tests.

The example below shows how **inline** can be deployed with **RcppGSL**. We implement the same column norm example, but this time as an R script which is compiled, linked and loaded on-the-fly. Compared to standard use of **inline**, we have to make sure to add a short section declaring which header files from **GSL** we need to use; the **RcppGSL** then communicates with **inline** to tell it about the location and names of libraries used to build code against **GSL**.

```
> require(inline)
> inctxt='
  #include <gsl/gsl_matrix.h>
  #include <gsl/gsl_blas.h>
'
> bodytxt='
RcppGSL::matrix<double> M = sM;      // create gsl data structures from SEXP
int k = M.ncol();
Rcpp::NumericVector n(k);            // to store results

for (int j = 0; j < k; j++) {
  RcppGSL::vector_view<double> colview = gsl_matrix_column (M, j);
  n[j] = gsl_blas_dnrm2(colview);
}
M.free() ;
return n;                            // return vector
'
> foo <- cxxfunction(
+   signature(sM="numeric"),
+   body=bodytxt, inc=inctxt, plugin="RcppGSL")
> ## see Section 8.4.13 of the GSL manual: create M as a sum of two outer products
> M <- outer(sin(0:9), rep(1,10), "*") + outer(rep(1, 10), cos(0:9), "*")
> foo(M)
[1] 4.314614 3.120504 2.193159 3.261141 2.534157 2.572810
[7] 4.204689 3.652017 2.085236 3.073134
```

The RcppGSL inline plugin supports creation of a package skeleton based on the inline function.

```
> package.skeleton( "mypackage", foo )
```

7 Summary

The GNU Scientific Library (GSL) by Galassi et al. (2010) offers a very comprehensive collection of rigorously developed and tested functions for applied scientific computing under a common Open Source license. This has lead to widespread deployment of **GSL** among a number of disciplines.

Using the automatic wrapping and converters offered by the **RcppGSL** package presented here, R users and programmers can now deploy algorithms provided by the **GSL** with greater ease.

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