

# Package ‘OptimalBinningWoE’

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**Type** Package

**Title** Optimal Binning and Weight of Evidence Framework for Modeling

**Version** 1.0.3

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**Description** High-performance implementation of 36 optimal binning algorithms (16 categorical, 20 numerical) for Weight of Evidence ('WoE') transformation, credit scoring, and risk modeling. Includes advanced methods such as Mixed Integer Linear Programming ('MILP'), Genetic Algorithms, Simulated Annealing, and Monotonic Regression. Features automatic method selection based on Information Value ('IV') maximization, strict monotonicity enforcement, and efficient handling of large datasets via 'Rcpp'. Fully integrated with the 'tidymodels' ecosystem for building robust machine learning pipelines. Based on methods described in Siddiqi (2006) <[doi:10.1002/9781119201731](https://doi.org/10.1002/9781119201731)> and Navas-Palencia (2020) <[doi:10.48550/arXiv.2001.08025](https://doi.org/10.48550/arXiv.2001.08025)>.

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**URL** <https://github.com/evandelton/OptimalBinningWoE>

**BugReports** <https://github.com/evandelton/OptimalBinningWoE/issues>

**Depends** R (>= 4.1.0)

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## Contents

.categorical_only_algorithms	3
.numerical_only_algorithms	4
.universal_algorithms	4
.valid_algorithms	5
bake.step_obwoe	5
control.obwoe	6
fit_logistic_regression	8
obcorr	10
obwoe	12
obwoe_algorithm	19
obwoe_algorithms	20
obwoe_apply	20
obwoe_bin_cutoff	22
obwoe_gains	23
obwoe_max_bins	28
obwoe_min_bins	29
ob_apply_woe_cat	30
ob_apply_woe_num	31
ob_categorical_cm	33
ob_categorical_dmv	37
ob_categorical_dp	41
ob_categorical_fetb	46
ob_categorical_gmb	51
ob_categorical_ivb	58
ob_categorical_jedi	63
ob_categorical_jedi_mwoe	72
ob_categorical_mba	80
ob_categorical_milp	88
ob_categorical_mob	90
ob_categorical_sab	93
ob_categorical_sbfp	96
ob_categorical_sketch	99
ob_categorical_swb	103
ob_categorical_udt	106
ob_cutpoints_cat	109
ob_cutpoints_num	111
ob_gains_table	112
ob_gains_table_feature	114
ob_numerical_bb	115

.categorical_only_algorithms	3
------------------------------	---

ob_numerical_cm	118
ob_numerical_dmv	121
ob_numerical_dp	123
ob_numerical_ewb	126
ob_numerical_fast_mdlp	129
ob_numerical_feth	132
ob_numerical_ir	134
ob_numerical_jedi	136
ob_numerical_jedi_mwoe	138
ob_numerical_kmb	141
ob_numerical_ldb	143
ob_numerical_lpdb	147
ob_numerical_mblp	150
ob_numerical_mdlp	155
ob_numerical_mob	162
ob_numerical_mrblp	168
ob_numerical_oslp	173
ob_numerical_sketch	177
ob_numerical_ubsd	181
ob_numerical_udt	185
ob_preprocess	188
plot.obwoe	192
plot.obwoe_gains	194
prep.step_obwoe	194
print.obwoe	195
print.step_obwoe	195
required_pkgs.step_obwoe	196
step_obwoe	196
summary.obwoe	201
tidy.step_obwoe	203
tunable.step_obwoe	204

<b>Index</b>	<b>205</b>
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---

.categorical\_only\_algorithms  
*Categorical-Only Algorithms*

---

## Description

Internal function returning algorithm identifiers that support only categorical features.

## Usage

.categorical\_only\_algorithms()

**Value**

A character vector of categorical-only algorithm names.

---

*.numerical\_only\_algorithms*

*Numerical-Only Algorithms*

---

**Description**

Internal function returning algorithm identifiers that support only numerical features.

**Usage**

`.numerical_only_algorithms()`

**Value**

A character vector of numerical-only algorithm names.

---

*.universal\_algorithms* *Universal Algorithms*

---

**Description**

Internal function returning algorithm identifiers that support both numerical and categorical features.

**Usage**

`.universal_algorithms()`

**Value**

A character vector of universal algorithm names.

---

.valid\_algorithms      *Valid Binning Algorithms*

---

### Description

Internal function returning the vector of all valid algorithm identifiers supported by the Optimal-BinningWoE package. Used for validation and parameter definition.

### Usage

```
.valid_algorithms()
```

### Value

A character vector of valid algorithm names including "auto".

---

bake.step\_obwoe      *Apply the Optimal Binning Transformation*

---

### Description

Applies the learned binning and WoE transformation to new data. This method is called by [bake](#) and should not be invoked directly.

### Usage

```
## S3 method for class 'step_obwoe'  
bake(object, new_data, ...)
```

### Arguments

object	A trained step_obwoe object.
new_data	A tibble or data frame to transform.
...	Additional arguments (currently unused).

### Value

A tibble with transformed columns according to the output parameter.

## Description

Constructs a validated list of control parameters for the `obwoe` master interface. These parameters govern the behavior of all supported binning algorithms, including convergence criteria, minimum bin sizes, and optimization limits.

## Usage

```
control.obwoe(
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  bin_separator = "%;%",
  verbose = FALSE,
  ...
)
```

## Arguments

<code>bin_cutoff</code>	Numeric value in (0, 1) specifying the minimum proportion of total observations that a bin must contain. Bins with fewer observations are merged with adjacent bins. Serves as a regularization mechanism to prevent overfitting and ensure statistical stability of WoE estimates. Recommended range: 0.02 to 0.10. Default is 0.05 (5%).
<code>max_n_prebins</code>	Integer specifying the maximum number of initial bins created before optimization. For high-cardinality categorical features, categories with similar event rates are pre-merged until this limit is reached. Higher values preserve more granularity but increase computational cost. Typical range: 10 to 50. Default is 20.
<code>convergence_threshold</code>	Numeric value specifying the tolerance for algorithm convergence. Iteration stops when the absolute change in Information Value between successive iterations falls below this threshold: $ IV_t - IV_{t-1}  < \epsilon$ . Smaller values yield more precise solutions at higher computational cost. Typical range: $10^{-4}$ to $10^{-8}$ . Default is $10^{-6}$ .
<code>max_iterations</code>	Integer specifying the maximum number of optimization iterations. Prevents infinite loops in degenerate cases. If the algorithm does not converge within this limit, it returns the best solution found. Typical range: 100 to 10000. Default is 1000.
<code>bin_separator</code>	Character string used to concatenate category names when multiple categories are merged into a single bin. Should be a string unlikely to appear in actual category names. Default is "%;%".

verbose	Logical indicating whether to print progress messages during feature processing. Useful for debugging or monitoring long-running jobs. Default is FALSE.
...	Additional named parameters reserved for algorithm-specific extensions. Currently unused but included for forward compatibility.

## Details

### Parameter Impact on Results:

**bin\_cutoff**: Lower values allow smaller bins, which may capture subtle patterns but risk unstable WoE estimates. The variance of WoE estimates increases as  $1/n_i$  where  $n_i$  is the bin size. For bins with fewer than ~30 observations, consider using Laplace or Bayesian smoothing (applied automatically by most algorithms).

**max\_n\_prebins**: Critical for categorical features with many levels. If a feature has 100 categories, setting `max_n_prebins = 20` will pre-merge similar categories into 20 groups before optimization.

**convergence\_threshold**: Trade-off between precision and speed. For exploratory analysis,  $10^{-4}$  is sufficient. For production models requiring reproducibility, use  $10^{-8}$  or smaller.

## Value

An S3 object of class "obwoe\_control" containing all specified parameters. This object is validated and can be passed directly to [obwoe](#).

## See Also

[obwoe](#) for the main binning interface.

## Examples

```
# Default control parameters
ctrl_default <- control.obwoe()
print(ctrl_default)

# Conservative settings for production
ctrl_production <- control.obwoe(
  bin_cutoff = 0.03,
  max_n_prebins = 30,
  convergence_threshold = 1e-8,
  max_iterations = 5000
)

# Aggressive settings for exploration
ctrl_explore <- control.obwoe(
  bin_cutoff = 0.01,
  max_n_prebins = 50,
  convergence_threshold = 1e-4,
  max_iterations = 500
)
```

---

**fit\_logistic\_regression**  
*Fit Logistic Regression Model*

---

## Description

This function fits a logistic regression model to binary classification data. It supports both dense and sparse matrix inputs for the predictor variables. The optimization is performed using the L-BFGS algorithm.

## Usage

```
fit_logistic_regression(X_r, y_r, maxit = 300L, eps_f = 1e-08, eps_g = 1e-05)
```

## Arguments

<code>X_r</code>	A numeric matrix or sparse matrix (dgCMatrix) of predictor variables. Rows represent observations and columns represent features.
<code>y_r</code>	A numeric vector of binary outcome values (0 or 1). Must have the same number of observations as rows in <code>X_r</code> .
<code>maxit</code>	Integer. Maximum number of iterations for the optimizer. Default is 300.
<code>eps_f</code>	Numeric. Convergence tolerance for the function value. Default is 1e-8.
<code>eps_g</code>	Numeric. Convergence tolerance for the gradient norm. Default is 1e-5.

## Details

The logistic regression model estimates the probability of the binary outcome  $y_i \in \{0, 1\}$  given predictors  $x_i$ :

$$P(y_i = 1|x_i) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip})}}$$

The function maximizes the log-likelihood:

$$\ell(\beta) = \sum_{i=1}^n [y_i \cdot (\beta^T x_i) - \ln(1 + e^{\beta^T x_i})]$$

Standard errors are computed from the inverse of the Hessian matrix evaluated at the estimated coefficients. Z-scores and p-values are derived under the assumption of asymptotic normality.

## Value

A list containing the results of the logistic regression fit:

`coefficients` Numeric vector of estimated regression coefficients.

`se` Numeric vector of standard errors for the coefficients.

`z_scores` Numeric vector of z-statistics for testing coefficient significance.

p\_values Numeric vector of p-values associated with the z-statistics.  
 loglikelihood Scalar. The maximized log-likelihood value.  
 gradient Numeric vector. The gradient at the solution.  
 hessian Matrix. The Hessian matrix evaluated at the solution.  
 convergence Logical. Whether the algorithm converged successfully.  
 iterations Integer. Number of iterations performed.  
 message Character. Convergence message.

### Note

- An intercept term is not automatically included. Users should add a column of ones to X\_r if an intercept is desired.
- If the Hessian matrix is singular (determinant is zero), standard errors, z-scores, and p-values will be returned as NA.
- The function uses the L-BFGS quasi-Newton optimization method.

### Examples

```

# Generate sample data
set.seed(123)
n <- 100
p <- 3
X <- matrix(rnorm(n * p), n, p)
# Add intercept column
X <- cbind(1, X)
colnames(X) <- c("(Intercept)", "X1", "X2", "X3")

# True coefficients
beta_true <- c(0.5, 1.2, -0.8, 0.3)

# Generate linear predictor
eta <- X %*% beta_true

# Generate binary outcome
prob <- 1 / (1 + exp(-eta))
y <- rbinom(n, 1, prob)

# Fit logistic regression
result <- fit_logistic_regression(X, y)

# View coefficients and statistics
print(data.frame(
  Coefficient = result$coefficients,
  Std_Error = result$se,
  Z_score = result$z_scores,
  P_value = result$p_values
))
# Check convergence
  
```

```
cat("Converged:", result$convergence, "\n")
cat("Log-Likelihood:", result$loglikelihood, "\n")
```

---

## obcorr

### *Compute Multiple Robust Correlations Between Numeric Variables*

---

#### Description

This function computes various correlation coefficients between all pairs of numeric variables in a data frame. It implements several classical and robust correlation measures, including Pearson, Spearman, Kendall, Hoeffding's D, Distance Correlation, Biweight Midcorrelation, and Percentage Bend correlation.

#### Usage

```
obcorr(df, method = "all", threads = 0L)
```

#### Arguments

df	A data frame containing numeric variables. Non-numeric columns will be automatically excluded. At least two numeric variables are required.
method	A character string specifying which correlation method(s) to compute. Possible values are: <ul style="list-style-type: none"> <li>• "all": Compute all available correlation methods (default).</li> <li>• "pearson": Compute only Pearson correlation.</li> <li>• "spearman": Compute only Spearman correlation.</li> <li>• "kendall": Compute only Kendall correlation.</li> <li>• "hoeffding": Compute only Hoeffding's D.</li> <li>• "distance": Compute only distance correlation.</li> <li>• "biweight": Compute only biweight midcorrelation.</li> <li>• "pbend": Compute only percentage bend correlation.</li> <li>• "robust": Compute robust correlations (biweight and pbend).</li> <li>• "alternative": Compute alternative correlations (hoeffding and distance).</li> </ul>
threads	An integer specifying the number of threads to use for parallel computation. If 0 (default), uses all available cores. Ignored if OpenMP is not available.

#### Details

The function supports multiple correlation methods simultaneously and utilizes OpenMP for parallel computation when available.

Available correlation methods:

- **Pearson**: Standard linear correlation coefficient.
- **Spearman**: Rank-based correlation coefficient.

- **Kendall**: Kendall's tau-b correlation coefficient.
- **Hoeffding**: Hoeffding's D statistic (scaled by 30).
- **Distance**: Distance correlation (Székely et al., 2007).
- **Biweight**: Biweight midcorrelation (robust alternative).
- **Pbend**: Percentage bend correlation (robust alternative).

### Value

A data frame with the following columns:

`x, y` Names of the variable pairs being correlated.  
`pearson` Pearson correlation coefficient.  
`spearman` Spearman rank correlation coefficient.  
`kendall` Kendall's tau-b correlation coefficient.  
`hoeffding` Hoeffding's D statistic (scaled).  
`distance` Distance correlation coefficient.  
`biweight` Biweight midcorrelation coefficient.  
`pbend` Percentage bend correlation coefficient.

The exact columns returned depend on the `method` parameter.

### Note

- Missing values (NA) are handled appropriately for each correlation method.
- For robust methods (biweight, pbend), fallback to Pearson correlation occurs when there are insufficient data points or numerical instability.
- Hoeffding's D requires at least 5 complete pairs.
- Distance correlation is computed without forming NxN distance matrices for memory efficiency.
- When OpenMP is available, computations are automatically parallelized across variable pairs.

### References

Székely, G.J., Rizzo, M.L., and Bakirov, N.K. (2007). Measuring and testing dependence by correlation of distances. *The Annals of Statistics*, 35(6), 2769-2794.

Wilcox, R.R. (1994). The percentage bend correlation coefficient. *Psychometrika*, 59(4), 601-616.

### Examples

```

# Create sample data
set.seed(123)
n <- 100
df <- data.frame(
  x1 = rnorm(n),
  x2 = rnorm(n),
  x3 = rt(n, df = 3), # Heavy-tailed distribution

```

```

x4 = sample(c(0, 1), n, replace = TRUE), # Binary variable
category = sample(letters[1:3], n, replace = TRUE) # Non-numeric column
)

# Add some relationships
df$x2 <- df$x1 + rnorm(n, 0, 0.5)
df$x3 <- df$x1^2 + rnorm(n, 0, 0.5)

# Compute all correlations
result_all <- obcorr(df)
head(result_all)

# Compute only robust correlations
result_robust <- obcorr(df, method = "robust")

# Compute only Pearson correlation with 2 threads
result_pearson <- obcorr(df, method = "pearson", threads = 2)

```

## Description

Master interface for optimal discretization and Weight of Evidence (WoE) computation across numerical and categorical predictors. This function serves as the primary entry point for the **Optimal-BinningWoE** package, providing automatic feature type detection, intelligent algorithm selection, and unified output structures for seamless integration into credit scoring and predictive modeling workflows.

## Usage

```

obwoe(
  data,
  target,
  feature = NULL,
  min_bins = 2,
  max_bins = 7,
  algorithm = "auto",
  control = control.obwoe()
)

```

## Arguments

data	A <code>data.frame</code> containing the predictor variables (features) and the response variable (target). All features to be binned must be present in this data frame. The data frame should not contain list-columns.
------	---

target	Character string specifying the column name of the response variable. Must be a binary outcome encoded as integers 0 (non-event) and 1 (event), or a multinomial outcome encoded as integers 0, 1, 2, ..., K. Missing values in the target are not permitted.
feature	Optional character vector specifying which columns to process. If NULL (default), all columns except target are processed. Features containing only missing values are automatically skipped with a warning.
min_bins	Integer specifying the minimum number of bins. Must satisfy $2 \leq \text{min\_bins} \leq \text{max\_bins}$ . Algorithms may produce fewer bins if the data has insufficient unique values. Default is 2.
max_bins	Integer specifying the maximum number of bins. Controls the granularity of discretization. Higher values capture more detail but risk overfitting. Typical values range from 5 to 10 for credit scoring applications. Default is 7.
algorithm	Character string specifying the binning algorithm. Use "auto" (default) for automatic selection based on target type: "jedi" for binary targets, "jedi_mwoe" for multinomial. See Details for the complete algorithm taxonomy.
control	A list of algorithm-specific control parameters created by <code>control.obwoe</code> . Provides fine-grained control over convergence thresholds, bin cutoffs, and other optimization parameters.

## Details

### Theoretical Foundation:

Weight of Evidence (WoE) transformation is a staple of credit scoring methodology, originating from information theory and the concept of evidential support (Good, 1950; Kullback, 1959). For a bin  $i$ , the WoE is defined as:

$$WoE_i = \ln \left( \frac{p_i}{n_i} \right) = \ln \left( \frac{N_{i,1}/N_1}{N_{i,0}/N_0} \right)$$

where:

- $N_{i,1}$  = number of events (target=1) in bin  $i$
- $N_{i,0}$  = number of non-events (target=0) in bin  $i$
- $N_1, N_0$  = total events and non-events, respectively
- $p_i = N_{i,1}/N_1$  = proportion of events in bin  $i$
- $n_i = N_{i,0}/N_0$  = proportion of non-events in bin  $i$

The Information Value (IV) quantifies the total predictive power of a binning:

$$IV = \sum_{i=1}^k (p_i - n_i) \times WoE_i = \sum_{i=1}^k (p_i - n_i) \times \ln \left( \frac{p_i}{n_i} \right)$$

where  $k$  is the number of bins. IV is equivalent to the Kullback-Leibler divergence between the event and non-event distributions.

### Algorithm Taxonomy:

The package provides 28 algorithms organized by supported feature types:

**Universal Algorithms** (both numerical and categorical):

<b>ID</b>	<b>Full Name</b>	<b>Method</b>
jedi	Joint Entropy-Driven Information	Heuristic + IV optimization
jedi_mwoe	JEDI Multinomial WoE	Extension for K>2 classes
cm	ChiMerge	Bottom-up chi-squared merging
dp	Dynamic Programming	Exact optimal IV partitioning
ddiv	Decision Tree MIV	Recursive partitioning
fetb	Fisher's Exact Test	Statistical significance-based
mob	Monotonic Optimal Binning	IV-optimal with monotonicity
sketch	Sketching	Probabilistic data structures
udt	Unsupervised Decision Tree	Entropy-based without target

**Numerical-Only Algorithms:**

<b>ID</b>	<b>Description</b>
bb	Branch and Bound (exact search)
ewb	Equal Width Binning (unsupervised)
fast_mdlp	Fast MDLP with pruning
ir	Isotonic Regression
kmb	K-Means Binning
ldb	Local Density Binning
lpdb	Local Polynomial Density
mblp	Monotonic Binning LP
mdlp	Minimum Description Length
mrb1p	Monotonic Regression LP
oslp	Optimal Supervised LP
ubsd	Unsupervised Std-Dev Based

**Categorical-Only Algorithms:**

<b>ID</b>	<b>Description</b>
gmb	Greedy Monotonic Binning
ivb	Information Value DP (exact)
mba	Modified Binning Algorithm
milp	Mixed Integer LP
sab	Simulated Annealing
sblp	Similarity-Based LP
swb	Sliding Window Binning

**Automatic Type Detection:**

Feature types are detected as follows:

- **Numerical:** numeric or integer vectors not of class `factor`
- **Categorical:** character, factor, or logical vectors

When `algorithm = "auto"`, the function selects:

- "jedi" for binary targets (recommended for most use cases)
- "jedi\_mwoe" for multinomial targets (K > 2 classes)

#### IV Interpretation Guidelines:

Siddiqi (2006) provides the following IV thresholds for variable selection:

IV Range	Predictive Power
< 0.02	Unpredictive
0.02 - 0.10	Weak
0.10 - 0.30	Medium
0.30 - 0.50	Strong
> 0.50	Suspicious (likely overfitting)

#### Computational Considerations:

Time complexity varies by algorithm:

- **JEDI, ChiMerge, MOB:**  $O(n \log n + k^2 m)$  where  $n$  = observations,  $k$  = bins,  $m$  = iterations
- **Dynamic Programming:**  $O(n \cdot k^2)$  for exact solution
- **Equal Width:**  $O(n)$  (fastest, but unsupervised)
- **MILP, SBLP:** Potentially exponential (NP-hard problems)

For large datasets ( $n > 10^6$ ), consider:

1. Using algorithm = "sketch" for approximate streaming
2. Reducing max\_n\_prebins via control.obwoe()
3. Sampling the data before binning

#### Value

An S3 object of class "obwoe" containing:

```
results Named list where each element contains the binning result for a single feature, including:
  bin Character vector of bin labels/intervals
  woe Numeric vector of Weight of Evidence per bin
  iv Numeric vector of Information Value contribution per bin
  count Integer vector of observation counts per bin
  count_pos Integer vector of positive (event) counts per bin
  count_neg Integer vector of negative (non-event) counts per bin
  cutpoints Numeric vector of bin boundaries (numerical only)
  converged Logical indicating algorithm convergence
  iterations Integer count of optimization iterations

  summary Data frame with one row per feature containing: feature (name), type (numerical/categorical),
  algorithm (used), n_bins (count), total_iv (sum), error (logical flag)

  target Name of the target column
  target_type Detected type: "binary" or "multinomial"
  n_features Number of features processed
  call The matched function call for reproducibility
```

## References

Good, I. J. (1950). Probability and the Weighing of Evidence. *Griffin, London*.

Kullback, S. (1959). Information Theory and Statistics. *Wiley, New York*.

Siddiqi, N. (2006). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. *John Wiley & Sons*. doi:[10.1002/9781119201731](https://doi.org/10.1002/9781119201731)

Thomas, L. C., Edelman, D. B., & Crook, J. N. (2002). Credit Scoring and Its Applications. *SIAM Monographs on Mathematical Modeling and Computation*. doi:[10.1137/1.9780898718317](https://doi.org/10.1137/1.9780898718317)

Navas-Palencia, G. (2020). Optimal Binning: Mathematical Programming Formulation and Solution Approach. *Expert Systems with Applications*, 158, 113508. doi:[10.1016/j.eswa.2020.113508](https://doi.org/10.1016/j.eswa.2020.113508)

Zeng, G. (2014). A Necessary Condition for a Good Binning Algorithm in Credit Scoring. *Applied Mathematical Sciences*, 8(65), 3229-3242.

## See Also

`control.obwoe` for algorithm-specific parameters, `obwoe_algorithms` to list all available algorithms with capabilities, `print.obwoe` for display methods, `ob_apply_woe_num` and `ob_apply_woe_cat` to apply WoE transformations to new data.

For individual algorithms with full parameter control: `ob_numerical_jedi`, `ob_categorical_jedi`, `ob_numerical_mdlp`, `ob_categorical_ivb`.

## Examples

```
# =====
# Example 1: Basic Usage with Mixed Feature Types
# =====
set.seed(42)
n <- 2000

# Simulate credit scoring data
df <- data.frame(
  # Numerical features
  age = pmax(18, pmin(80, rnorm(n, 45, 15))),
  income = exp(rnorm(n, 10, 0.8)),
  debt_ratio = rbeta(n, 2, 5),
  credit_history_months = rpois(n, 60),

  # Categorical features
  education = sample(c("High School", "Bachelor", "Master", "PhD"),
    n,
    replace = TRUE, prob = c(0.35, 0.40, 0.20, 0.05)
  ),
  employment = sample(c("Employed", "Self-Employed", "Unemployed", "Retired"),
    n,
    replace = TRUE, prob = c(0.60, 0.20, 0.10, 0.10)
  ),

  # Binary target (default probability varies by features)
  target = rbinom(n, 1, 0.15)
)
```

```
# Process all features with automatic algorithm selection
result <- obwoe(df, target = "target")
print(result)

# View detailed summary
print(result$summary)

# Access results for a specific feature
age_bins <- result$results$age
print(data.frame(
  bin = age_bins$bin,
  woe = round(age_bins$woe, 3),
  iv = round(age_bins$iv, 4),
  count = age_bins$count
))

# =====
# Example 2: Using a Specific Algorithm
# =====

# Use MDLP for numerical features (entropy-based)
result_mdlp <- obwoe(df,
  target = "target",
  feature = c("age", "income"),
  algorithm = "mdlp",
  min_bins = 3,
  max_bins = 6
)

cat("\nMDLP Results:\n")
print(result_mdlp$summary)

# =====
# Example 3: Custom Control Parameters
# =====

# Fine-tune algorithm behavior
ctrl <- control.obwoe(
  bin_cutoff = 0.02, # Minimum 2% per bin
  max_n_prebins = 30, # Allow more initial bins
  convergence_threshold = 1e-8
)

result_custom <- obwoe(df,
  target = "target",
  feature = "debt_ratio",
  algorithm = "jedi",
  control = ctrl
)

cat("\nCustom JEDI Result:\n")
print(result_custom$results$debt_ratio$bin)
```

```

# =====
# Example 4: Comparing Multiple Algorithms
# =====

algorithms <- c("jedi", "mdlp", "ewb", "mob")
iv_comparison <- sapply(algorithms, function(algo) {
  tryCatch(
    {
      res <- obwoe(df, target = "target", feature = "income", algorithm = algo)
      res$summary$total_iv
    },
    error = function(e) NA_real_
  )
})

cat("\nAlgorithm Comparison (IV for 'income'):\n")
print(sort(iv_comparison, decreasing = TRUE))

# =====
# Example 5: Feature Selection Based on IV
# =====

# Process all features and select those with IV > 0.02
result_all <- obwoe(df, target = "target")

strong_features <- result_all$summary[
  result_all$summary$total_iv >= 0.02 & !result_all$summary$error,
  c("feature", "total_iv", "n_bins")
]
strong_features <- strong_features[order(-strong_features$total_iv), ]

cat("\nFeatures with IV >= 0.02 (predictive):\n")
print(strong_features)

# =====
# Example 6: Handling Algorithm Compatibility
# =====

# MDLP only works for numerical - will fail for categorical
result_mixed <- obwoe(df,
  target = "target",
  algorithm = "mdlp"
)

# Check for errors
cat("\nCompatibility check:\n")
print(result_mixed$summary[, c("feature", "type", "error")])

```

---

obwoe_algorithm	<i>Binning Algorithm Parameter</i>
-----------------	------------------------------------

---

## Description

A qualitative tuning parameter for selecting the optimal binning algorithm in [step\\_obwoe](#).

## Usage

```
obwoe_algorithm(values = NULL)
```

## Arguments

**values** A character vector of algorithm names to include in the parameter space. If `NULL` (default), includes all 29 algorithms (28 specific algorithms plus "auto").

## Details

The algorithms are organized into three groups:

**Universal** (support both numerical and categorical features): "auto", "jedi", "jedi\_mwoe", "cm", "dp", "dmiv", "fetb", "mob", "sketch", "udt"

**Numerical only**: "bb", "ewb", "fast\_mdlp", "ir", "kmb", "ldb", "lpdb", "mblp", "mdlp", "mrblp", "oslp", "ubsd"

**Categorical only**: "gmb", "ivb", "mba", "milp", "sab", "sblp", "swb"

When tuning with mixed feature types, consider restricting `values` to universal algorithms only.

## Value

A `dials` qualitative parameter object.

## See Also

[step\\_obwoe](#), [obwoe](#)

## Examples

```
# Default: all algorithms
obwoe_algorithm()

# Restrict to universal algorithms for mixed data
obwoe_algorithm(values = c("jedi", "mob", "dp", "cm"))

# Numerical-only algorithms
obwoe_algorithm(values = c("mdlp", "fast_mdlp", "ewb", "ir"))
```

---

obwoe_algorithms	<i>List Available Algorithms</i>
------------------	----------------------------------

---

## Description

Returns a data frame with all available binning algorithms.

## Usage

```
obwoe_algorithms()
```

## Value

A data frame with algorithm information.

## Examples

```
obwoe_algorithms()
```

---

obwoe_apply	<i>Apply Weight of Evidence Transformations to New Data</i>
-------------	---

---

## Description

Applies the binning and Weight of Evidence (WoE) transformations learned by [obwoe](#) to new data. This is the scoring function for deploying WoE-based models in production. For each feature, the function assigns observations to bins and maps them to their corresponding WoE values.

## Usage

```
obwoe_apply(
  data,
  obj,
  suffix_bin = "_bin",
  suffix_woe = "_woe",
  keep_original = TRUE,
  na_woe = 0
)
```

## Arguments

data	A <code>data.frame</code> containing the features to transform. Must include all features present in the <code>obj</code> results. The target column is optional; if present, it will be included in the output.
obj	An object of class "obwoe" returned by <code>obwoe</code> .
suffix_bin	Character string suffix for bin columns. Default is "_bin".
suffix_woe	Character string suffix for WoE columns. Default is "_woe".
keep_original	Logical. If TRUE (default), include the original feature columns in the output. If FALSE, only bin and WoE columns are returned.
na_woe	Numeric value to assign when an observation cannot be mapped to a bin (e.g., new categories not seen during training). Default is 0.

## Details

### Bin Assignment Logic:

**Numerical Features:** Observations are assigned to bins based on cutpoints stored in the `obwoe` object. The `cut()` function is used with intervals  $(a_i, a_{i+1}]$  where  $a_0 = -\infty$  and  $a_k = +\infty$ .

**Categorical Features:** Categories are matched directly to bin labels. Categories not seen during training are assigned NA for bin and `na_woe` for WoE.

### Production Deployment:

For production scoring, it is recommended to:

1. Train the binning model using `obwoe()` on the training set
2. Save the fitted object with `saveRDS()`
3. Load and apply using `obwoe_apply()` on new data

The WoE-transformed features can be used directly as inputs to logistic regression or other linear models, enabling interpretable credit scorecards.

## Value

A `data.frame` containing:

`target` The target column (if present in `data`)  
`<feature>` Original feature values (if `keep_original` = TRUE)  
`<feature>_bin` Assigned bin label for each observation  
`<feature>_woe` Weight of Evidence value for the assigned bin

## References

Siddiqi, N. (2006). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. *John Wiley & Sons*. doi:10.1002/9781119201731

## See Also

`obwoe` for fitting the binning model, `summary.obwoe` for model diagnostics.

## Examples

```

# =====
# Example 1: Basic Usage - Train and Apply
# =====
set.seed(42)
n <- 1000

# Training data
train_df <- data.frame(
  age = rnorm(n, 40, 15),
  income = exp(rnorm(n, 10, 0.8)),
  education = sample(c("HS", "BA", "MA", "PhD"), n, replace = TRUE),
  target = rbinom(n, 1, 0.15)
)

# Fit binning model
model <- obwoe(train_df, target = "target")

# New data for scoring (could be validation/test set)
new_df <- data.frame(
  age = c(25, 45, 65),
  income = c(20000, 50000, 80000),
  education = c("HS", "MA", "PhD")
)

# Apply transformations
scored <- obwoe_apply(new_df, model)
print(scored)

# Use WoE features for downstream modeling
woe_cols <- grep("_woe$", names(scored), value = TRUE)
print(woe_cols)

# =====
# Example 2: Without Original Features
# =====

scored_compact <- obwoe_apply(new_df, model, keep_original = FALSE)
print(scored_compact)

```

---

obwoe\_bin\_cutoff      *Bin Cutoff Parameter*

---

## Description

A quantitative tuning parameter for the minimum bin support (proportion of observations per bin) in [step\\_obwoe](#).

## Usage

```
obwoe_bin_cutoff(range = c(0.01, 0.1), trans = NULL)
```

## Arguments

range	A two-element numeric vector specifying the minimum and maximum values for the parameter. Default is c(0.01, 0.10).
trans	A transformation object from the scales package, or NULL for no transformation. Default is NULL.

## Details

The bin cutoff specifies the minimum proportion of observations that each bin must contain. Bins with fewer observations are merged with adjacent bins. This serves as a regularization mechanism:

- Lower values (e.g., 0.01) allow smaller bins, capturing subtle patterns but risking unstable WoE estimates.
- Higher values (e.g., 0.10) enforce larger bins, producing more stable estimates but potentially missing important patterns.

For credit scoring, values between 0.02 and 0.05 are typical. Regulatory guidelines often require minimum bin sizes for model stability.

## Value

A dials quantitative parameter object.

## See Also

[step\\_obwoe](#), [control.obwoe](#)

## Examples

```
obwoe_bin_cutoff()  
obwoe_bin_cutoff(range = c(0.02, 0.08))
```

## Description

Computes a comprehensive gains table (also known as a lift table or decile analysis) for evaluating the discriminatory power of credit scoring models and optimal binning transformations. The gains table is a fundamental tool in credit risk management for model validation, cutoff selection, and regulatory reporting (Basel II/III, IFRS 9).

This function accepts three types of input:

1. An "obwoe" object from [obwoe](#) (uses stored binning)
2. A `data.frame` from [obwoe\\_apply](#) (uses bin/WoE columns)
3. Any `data.frame` with a grouping variable (e.g., score deciles)

## Usage

```
obwoe_gains(
  obj,
  target = NULL,
  feature = NULL,
  use_column = c("auto", "bin", "woe", "direct"),
  sort_by = c("id", "woe", "event_rate", "bin"),
  n_groups = NULL
)
```

## Arguments

<code>obj</code>	Input object: an "obwoe" object, a <code>data.frame</code> from <a href="#">obwoe_apply</a> , or any <code>data.frame</code> containing a grouping variable and target values.
<code>target</code>	Integer vector of binary target values (0/1) or the name of the target column in <code>obj</code> . Required for <code>data.frame</code> inputs. For "obwoe" objects, the target is extracted automatically.
<code>feature</code>	Character string specifying the feature/variable to analyze. For "obwoe" objects: defaults to the feature with highest IV. For <code>data.frame</code> objects: can be any column name representing groups (e.g., "age_bin", "age_woe", "score_decile").
<code>use_column</code>	Character string specifying which column type to use when <code>obj</code> is a <code>data.frame</code> from <a href="#">obwoe_apply</a> :
	"bin" Use the <feature>_bin column (default)
	"woe" Use the <feature>_woe column (groups by WoE values)
	"auto" Automatically detect: use _bin if available
	"direct" Use the feature column name directly (for any variable)
<code>sort_by</code>	Character string specifying sort order for bins:
	"woe" Descending WoE (highest risk first) - default
	"event_rate" Descending event rate
	"bin" Alphabetical/natural order
<code>n_groups</code>	Integer. For continuous variables (e.g., scores), the number of groups (deciles) to create. Default is <code>NULL</code> (use existing groups). Set to 10 for standard decile analysis.

## Details

### Gains Table Construction:

The gains table is constructed by:

1. Sorting observations by risk score or WoE (highest risk first)
2. Grouping into bins (pre-defined or created via quantiles)
3. Computing bin-level and cumulative statistics

The table enables assessment of model rank-ordering ability: a well-calibrated model should show monotonically increasing event rates as risk score increases.

### Bin-Level Statistics (18 metrics):

Column	Formula	Description
bin	-	Bin label or interval
count	$n_i$	Total observations in bin
count_pct	$n_i/N$	Proportion of total population
pos_count	$n_{i,1}$	Event count (Bad, target=1)
neg_count	$n_{i,0}$	Non-event count (Good, target=0)
pos_rate	$n_{i,1}/n_i$	Event rate (Bad rate) in bin
neg_rate	$n_{i,0}/n_i$	Non-event rate (Good rate)
pos_pct	$n_{i,1}/N_1$	Distribution of events
neg_pct	$n_{i,0}/N_0$	Distribution of non-events
odds	$n_{i,1}/n_{i,0}$	Odds of event
log_odds	$\ln(\text{odds})$	Log-odds (logit)
woe	$\ln(p_i/q_i)$	Weight of Evidence
iv	$(p_i - q_i) \cdot WoE_i$	Information Value contribution
cum_pos_pct	$\sum_{j \leq i} p_j$	Cumulative events captured
cum_neg_pct	$\sum_{j \leq i} q_j$	Cumulative non-events
ks	$ F_1(i) - F_0(i) $	KS statistic at bin
lift	$\text{pos\_rate}/\bar{p}$	Lift over random
capture_rate	$\text{cum\_pos\_pct}$	Cumulative capture rate

### Global Performance Metrics:

**Kolmogorov-Smirnov (KS) Statistic:** Maximum absolute difference between cumulative distributions of events and non-events. Measures the model's ability to separate populations.

$$KS = \max_i |F_1(i) - F_0(i)|$$

KS Range	Interpretation
< 20%	Poor discrimination
20-40%	Acceptable
40-60%	Good
60-75%	Very good
> 75%	Excellent (verify for data leakage)

**Gini Coefficient:** Measure of inequality between event and non-event distributions. Equivalent to  $2 \cdot \text{AUC} - 1$ , representing the area between the Lorenz curve and the line of equality.

$$Gini = 2 \times AUC - 1$$

**Area Under ROC Curve (AUC):** Probability that a randomly chosen event is ranked higher than a randomly chosen non-event. Computed via the trapezoidal rule.

**Total Information Value (IV):** Sum of IV contributions across all bins. See [obwoe](#) for interpretation guidelines.

#### Use Cases:

**Model Validation:** Verify rank-ordering (monotonic event rates) and acceptable KS/Gini.

**Cutoff Selection:** Identify the bin where the model provides optimal separation for business rules (e.g., auto-approve above score X).

**Population Stability:** Compare gains tables over time to detect model drift.

**Regulatory Reporting:** Generate metrics required by Basel II/III and IFRS 9 frameworks.

#### Value

An S3 object of class "obwoe\_gains" containing:

table Data frame with 18 statistics per bin (see Details)

metrics Named list of global performance metrics:

ks Kolmogorov-Smirnov statistic (%)

gini Gini coefficient (%)

auc Area Under ROC Curve

total\_iv Total Information Value

ks\_bin Bin where maximum KS occurs

feature Feature/variable name analyzed

n\_bins Number of bins/groups

n\_obs Total observations

event\_rate Overall event rate

#### References

Siddiqi, N. (2006). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. *John Wiley & Sons*. [doi:10.1002/9781119201731](#)

Thomas, L. C., Edelman, D. B., & Crook, J. N. (2002). Credit Scoring and Its Applications. *SIAM Monographs on Mathematical Modeling and Computation*. [doi:10.1137/1.9780898718317](#)

Anderson, R. (2007). The Credit Scoring Toolkit: Theory and Practice for Retail Credit Risk Management. *Oxford University Press*.

Hand, D. J., & Henley, W. E. (1997). Statistical Classification Methods in Consumer Credit Scoring: A Review. *Journal of the Royal Statistical Society: Series A*, 160(3), 523-541. [doi:10.1111/j.1467-985X.1997.00078.x](#)

#### See Also

[obwoe](#) for optimal binning, [obwoe\\_apply](#) for scoring new data, [plot.obwoe\\_gains](#) for visualization (cumulative gains, KS, lift).

## Examples

```

# =====
# Example 1: From obwoe Object (Standard Usage)
# =====
set.seed(42)
n <- 1000
df <- data.frame(
  age = rnorm(n, 40, 15),
  income = exp(rnorm(n, 10, 0.8)),
  score = rnorm(n, 600, 100),
  target = rbinom(n, 1, 0.15)
)

model <- obwoe(df, target = "target")
gains <- obwoe_gains(model, feature = "age")
print(gains)

# Access metrics
cat("KS:", gains$metrics$ks, "%\n")
cat("Gini:", gains$metrics$gini, "%\n")

# =====
# Example 2: From obwoe_apply Output - Using Bin Column
# =====
scored <- obwoe_apply(df, model)

# Default: uses age_bin column
gains_bin <- obwoe_gains(scored,
  target = df$target, feature = "age",
  use_column = "bin"
)

# =====
# Example 3: From obwoe_apply Output - Using WoE Column
# =====
# Group by WoE values (continuous analysis)
gains_woe <- obwoe_gains(scored,
  target = df$target, feature = "age",
  use_column = "woe", n_groups = 5
)

# =====
# Example 4: Any Variable - Score Decile Analysis
# =====
# Create score deciles manually
df$score_decile <- cut(df$score,
  breaks = quantile(df$score, probs = seq(0, 1, 0.1)),
  include.lowest = TRUE, labels = 1:10
)

# Analyze score deciles directly
gains_score <- obwoe_gains(df,

```

```

target = "target", feature = "score_decile",
use_column = "direct"
)
print(gains_score)

# =====
# Example 5: Automatic Decile Creation
# =====
# Use n_groups to automatically create quantile groups
gains_auto <- obwoe_gains(df,
  target = "target", feature = "score",
  use_column = "direct", n_groups = 10
)

```

---

obwoe_max_bins	<i>Maximum Bins Parameter</i>
----------------	-------------------------------

---

## Description

A quantitative tuning parameter for the maximum number of bins in [step\\_obwoe](#).

## Usage

```
obwoe_max_bins(range = c(5L, 20L), trans = NULL)
```

## Arguments

range	A two-element integer vector specifying the minimum and maximum values for the parameter. Default is c(5L, 20L).
trans	A transformation object from the scales package, or NULL for no transformation. Default is NULL.

## Details

The maximum number of bins limits algorithm complexity and helps prevent overfitting. Higher values allow more granular discretization but may capture noise rather than signal.

For credit scoring applications, max\_bins is typically set between 5 and 10 to balance predictive power with interpretability. Values above 15 are rarely necessary and may indicate overfitting.

## Value

A dials quantitative parameter object.

## See Also

[step\\_obwoe](#), [obwoe\\_min\\_bins](#)

## Examples

```
obwoe_max_bins()  
obwoe_max_bins(range = c(4L, 12L))
```

---

obwoe_min_bins	<i>Minimum Bins Parameter</i>
----------------	-------------------------------

---

## Description

A quantitative tuning parameter for the minimum number of bins in [step\\_obwoe](#).

## Usage

```
obwoe_min_bins(range = c(2L, 5L), trans = NULL)
```

## Arguments

**range** A two-element integer vector specifying the minimum and maximum values for the parameter. Default is `c(2L, 5L)`.

**trans** A transformation object from the `scales` package, or `NULL` for no transformation. Default is `NULL`.

## Details

The minimum number of bins constrains the algorithm to create at least this many bins. Setting `min_bins = 2` allows maximum flexibility, while higher values ensure more granular discretization.

For credit scoring applications, `min_bins` is typically set between 2 and 4 to avoid forcing artificial splits on weakly predictive variables.

## Value

A `dials` quantitative parameter object.

## See Also

[step\\_obwoe](#), [obwoe\\_max\\_bins](#)

## Examples

```
obwoe_min_bins()  
obwoe_min_bins(range = c(3L, 7L))
```

---

ob_apply_woe_cat	<i>Apply Optimal Weight of Evidence (WoE) to a Categorical Feature</i>
------------------	--

---

## Description

Transforms a categorical feature into its corresponding Weight of Evidence (WoE) values using pre-computed binning results from an optimal binning algorithm (e.g., [ob\\_categorical\\_cm](#)).

## Usage

```
ob_apply_woe_cat(
  obresults,
  feature,
  bin_separator = "%;%",
  missing_values = c("NA", "Missing", ""))
)
```

## Arguments

obresults	List output from an optimal binning function for categorical variables. Must contain elements <code>bin</code> (character vector of bin labels) and <code>woe</code> (numeric vector of WoE values). Bins may represent individual categories or merged groups separated by <code>bin_separator</code> .
feature	Character or factor vector of categorical values to be transformed. Automatically coerced to character if provided as factor.
bin_separator	Character string used to separate multiple categories within a single bin label (default: "%;%"). For example, a bin "A%;B%;C%" contains categories A, B, and C.
missing_values	Character vector specifying which values should be treated as missing (default: c("NA", "Missing", "")). These values are matched against a special bin labeled "NA" or "Missing" in <code>obresults</code> .

## Details

This function is typically used in a two-step workflow:

1. Train binning on training data: `bins <- ob_categorical_cm(feature_train, target_train)`
2. Apply WoE to new data: `woe_test <- ob_apply_woe_cat(bins, feature_test)`

The function performs exact string matching between categories in `feature` and the bin labels in `obresults$bin`. For merged bins (containing `bin_separator`), the string is split and each component is matched individually.

## Value

Numeric vector of WoE values with the same length as `feature`. Categories not found in `obresults` will produce NA values with a warning.

## Examples

```

# Mock data
train_data <- data.frame(
  category = c("A", "B", "A", "C", "B", "A"),
  default = c(0, 1, 0, 1, 0, 0)
)
test_data <- data.frame(
  category = c("A", "C", "B")
)

# Train binning on training set
train_bins <- ob_categorical_cm(
  feature = train_data$category,
  target = train_data$default
)

# Apply to test set
test_woe <- ob_apply_woe_cat(
  obresults = train_bins,
  feature = test_data$category
)

# Handle custom missing indicators
test_woe <- ob_apply_woe_cat(
  obresults = train_bins,
  feature = test_data$category,
  missing_values = c("NA", "Unknown", "N/A", ""))

```

---

ob\_apply\_woe\_num

*Apply Optimal Weight of Evidence (WoE) to a Numerical Feature*

---

## Description

Transforms a numerical feature into its corresponding Weight of Evidence (WoE) values using pre-computed binning results from an optimal binning algorithm (e.g., [ob\\_numerical\\_mdlp](#), [ob\\_numerical\\_mob](#)).

## Usage

```

ob_apply_woe_num(
  obresults,
  feature,
  include_upper_bound = TRUE,
  missing_values = c(-999)
)

```

## Arguments

<code>obresults</code>	List output from an optimal binning function for numerical variables. Must contain elements <code>cutpoints</code> (numeric vector of bin boundaries) and <code>woe</code> (numeric vector of WoE values). The number of WoE values should equal <code>length(cutpoints) + 1</code> .
<code>feature</code>	Numeric vector of values to be transformed. Automatically coerced to numeric if provided in another type.
<code>include_upper_bound</code>	Logical flag controlling interval boundary behavior (default: TRUE): <ul style="list-style-type: none"> <li>• TRUE: Intervals are <math>(lower, upper]</math> (right-closed).</li> <li>• FALSE: Intervals are <math>[lower, upper)</math> (left-closed).</li> </ul> This must match the convention used during binning.
<code>missing_values</code>	Numeric vector of values to be treated as missing (default: <code>c(-999)</code> ). These values are assigned the WoE of the special missing bin if it exists in <code>obresults</code> , or NA otherwise.

## Details

This function is typically used in a two-step workflow:

1. Train binning on training data: `bins <- ob_numerical_mdlp(feature_train, target_train)`
2. Apply WoE to new data: `woe_test <- ob_apply_woe_num(bins, feature_test)`

**Bin Assignment Logic:** For  $k$  cutpoints  $c_1 < c_2 < \dots < c_k$ , values are assigned as:

- Bin 1:  $x \leq c_1$  (if `include_upper_bound = TRUE`)
- Bin  $i$ :  $c_{i-1} < x \leq c_i$  for  $i = 2, \dots, k$
- Bin  $k+1$ :  $x > c_k$

### Handling of Edge Cases:

- Values in `missing_values` are matched against a bin labeled "NA" or "Missing" in `obresults$bin` (if available).
- `Inf` and `-Inf` are assigned to the last and first bins, respectively.
- Values exactly equal to `cutpoints` follow the `include_upper_bound` convention.

## Value

Numeric vector of WoE values with the same length as `feature`. Values outside the range of `cutpoints` are assigned to the first or last bin. NA values in `feature` are propagated to the output unless explicitly listed in `missing_values`.

## See Also

[ob\\_numerical\\_mdlp](#) for MDLP binning, [ob\\_numerical\\_mob](#) for monotonic binning, [ob\\_apply\\_woe\\_cat](#) for applying WoE to categorical features.

## Examples

```

# Mock data
train_data <- data.frame(
  income = c(50000, 75000, 30000, 45000, 80000, 60000),
  default = c(0, 0, 1, 1, 0, 0)
)
test_data <- data.frame(
  income = c(55000, 35000, 90000)
)

# Train binning on training set
train_bins <- ob_numerical_mdlp(
  feature = train_data$income,
  target = train_data$default
)

# Apply to test set
test_woe <- ob_apply_woe_num(
  obresults = train_bins,
  feature = test_data$income
)

# Handle custom missing indicators (e.g., -999, -1)
test_woe <- ob_apply_woe_num(
  obresults = train_bins,
  feature = test_data$income,
  missing_values = c(-999, -1, -9999)
)

# Use left-closed intervals (match scikit-learn convention)
test_woe <- ob_apply_woe_num(
  obresults = train_bins,
  feature = test_data$income,
  include_upper_bound = FALSE
)

```

---

## Description

Performs supervised discretization of categorical variables using an enhanced implementation of the ChiMerge algorithm (Kerber, 1992) with optional Chi2 extension (Liu & Setiono, 1995). This method optimally groups categorical levels based on their relationship with a binary target variable to maximize predictive power while maintaining statistical significance.

**Usage**

```
ob_categorical_cm(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  chi_merge_threshold = 0.05,
  use_chi2_algorithm = FALSE
)
```

**Arguments**

feature	A character vector or factor representing the categorical predictor variable to be binned.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. Threshold for treating categories as rare. Categories with frequency $< \text{bin\_cutoff}$ will be merged with their most similar neighbors. Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. Maximum number of initial pre-bins before merging. Controls computational complexity. Must be $\geq 2$ . Defaults to 20.
bin_separator	String. Separator used when combining multiple categories into a single bin label. Defaults to "%;%".
convergence_threshold	Numeric. Convergence tolerance for iterative merging process. Smaller values require stricter convergence. Must be $> 0$ . Defaults to 1e-6.
max_iterations	Integer. Maximum iterations for the merging algorithm. Prevents infinite loops. Must be $> 0$ . Defaults to 1000.
chi_merge_threshold	Numeric. Statistical significance level (p-value) for chi-square tests during merging. Higher values create fewer bins. Value must be in (0, 1). Defaults to 0.05.
use_chi2_algorithm	Logical. If TRUE, uses the Chi2 variant which performs multi-pass merging with decreasing significance thresholds. Defaults to FALSE.

**Details**

The algorithm implements two main approaches:

1. Standard ChiMerge: Iteratively merges adjacent bins with lowest chi-square statistics until all remaining pairs are statistically distinguishable at the specified significance level.
2. Chi2 Algorithm (when `use_chi2_algorithm = TRUE`): Performs multiple passes with decreasing significance thresholds ( $0.5 \rightarrow 0.001$ ), creating more robust binning structures particularly for noisy data.

Key features include:

- Rare category handling through pre-merging
- Monotonicity enforcement of Weight of Evidence
- Numerical stability with underflow protection
- Efficient chi-square caching for performance
- Comprehensive input validation and error handling

Information Value interpretation:

- $< 0.02$ : Predictive power not useful
- $0.02-0.1$ : Weak predictive power
- $0.1-0.3$ : Medium predictive power
- $0.3-0.5$ : Strong predictive power
- $> 0.5$ : Suspiciously high (potential overfitting)

## Value

A list containing binning results with the following components:

- `id`: Integer vector of bin identifiers (1:n\_bins)
- `bin`: Character vector of bin labels (merged category names)
- `woe`: Numeric vector of Weight of Evidence for each bin
- `iv`: Numeric vector of Information Value contribution per bin
- `count`: Integer vector of total observations per bin
- `count_pos`: Integer vector of positive cases per bin
- `count_neg`: Integer vector of negative cases per bin
- `converged`: Logical indicating if algorithm converged
- `iterations`: Integer count of algorithm iterations performed
- `algorithm`: Character string identifying algorithm used
- `warnings`: Character vector of any warnings encountered
- `metadata`: List with additional diagnostic information:
  - `total_iv`: Total Information Value of the binned variable
  - `n_bins`: Final number of bins produced
  - `unique_categories`: Number of unique input categories
  - `total_obs`: Total number of observations processed
  - `execution_time_ms`: Processing time in milliseconds
  - `monotonic`: Direction of WoE monotonicity ("increasing"/"decreasing")

## Author(s)

Developed as part of the OptimalBinningWoE package

## References

Kerber, R. (1992). ChiMerge: Discretization of numeric attributes. In *Proceedings of the Tenth National Conference on Artificial Intelligence* (pp. 123-128).

Liu, B., & Setiono, R. (1995). Chi2: Feature selection and discretization of numeric attributes. In *Proceedings of the Seventh IEEE International Conference on Tools with Artificial Intelligence* (pp. 372-377).

## Examples

```
# Example 1: Basic usage with synthetic data
set.seed(123)
n <- 1000
categories <- c("A", "B", "C", "D", "E", "F", "G", "H")
feature <- sample(categories, n, replace = TRUE, prob = c(
  0.2, 0.15, 0.15,
  0.1, 0.1, 0.1,
  0.1, 0.1
))
# Create target with some association to categories
probs <- c(0.3, 0.4, 0.5, 0.6, 0.7, 0.75, 0.8, 0.85) # increasing probability
target <- sapply(seq_along(feature), function(i) {
  cat_idx <- which(categories == feature[i])
  rbinom(1, 1, probs[cat_idx])
})

result <- ob_categorical_cm(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# View metadata
print(paste("Total IV:", round(result$metadata$total_iv, 3)))
print(paste("Algorithm converged:", result$converged))

# Example 2: Using Chi2 algorithm for more conservative binning
result_chi2 <- ob_categorical_cm(feature, target,
  use_chi2_algorithm = TRUE,
  max_bins = 6
)

# Compare number of bins
cat("Standard ChiMerge bins:", result$metadata$n_bins, "\n")
cat("Chi2 algorithm bins:", result_chi2$metadata$n_bins, "\n")
```

---

ob_categorical_dmv	<i>Optimal Binning for Categorical Variables using Divergence Measures</i>
--------------------	--

---

## Description

Performs supervised discretization of categorical variables using a divergence-based hierarchical merging algorithm. This implementation supports multiple information-theoretic and metric divergence measures as described by Zeng (2013), enabling flexible optimization of binning structures for credit scoring and binary classification tasks.

## Usage

```
ob_categorical_dmv(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  bin_method = "woe1",
  divergence_method = "l2"
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Missing values are not permitted.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 2$ . If the final number of bins after merging falls below this threshold, the algorithm will attempt to split bins. Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm performs hierarchical merging until this constraint is satisfied. Defaults to 5.
bin_cutoff	Numeric. Frequency threshold for rare category handling. Categories with relative frequency below this value are candidates for pre-binning. Must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. Maximum number of initial bins before the main merging phase. When unique categories exceed this limit, rare categories are pre-merged into an "other" bin. Must be $\geq 2$ . Defaults to 20.

<code>bin_separator</code>	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
<code>convergence_threshold</code>	Numeric. Convergence tolerance for the iterative merging process. Merging stops when the change in minimum divergence between iterations falls below this threshold. Must be > 0. Defaults to 1e-6.
<code>max_iterations</code>	Integer. Maximum number of merge operations allowed. Prevents infinite loops in edge cases. Must be > 0. Defaults to 1000.
<code>bin_method</code>	Character string specifying the Weight of Evidence calculation method. Must be one of: $\text{"woe"} \text{ Traditional WoE: } \ln \left( \frac{p_i/P}{n_i/N} \right)$ $\text{"woe1"} \text{ Smoothed WoE (Zeng): } \ln \left( \frac{g_i+0.5}{b_i+0.5} \right)$ The smoothed variant provides numerical stability for sparse bins. Defaults to "woe1".
<code>divergence_method</code>	Character string specifying the divergence measure used for determining bin similarity. Must be one of: $\text{"he"} \text{ Hellinger Distance: } \sum (\sqrt{p_i} - \sqrt{n_i})^2$ $\text{"k1"} \text{ Symmetrized Kullback-Leibler Divergence}$ $\text{"k1j"} \text{ Jeffreys J-Divergence: } (p - n) \ln(p/n)$ $\text{"tr"} \text{ Triangular Discrimination: } (p - n)^2 / (p + n)$ $\text{"sc"} \text{ Symmetric Chi-Square: } (p - n)^2 (p + n) / (pn)$ $\text{"js"} \text{ Jensen-Shannon Divergence}$ $\text{"l1"} \text{ L1 Metric (Manhattan Distance): }  p - n $ $\text{"l2"} \text{ L2 Metric (Euclidean Distance): } \sqrt{\sum (p - n)^2}$ $\text{"ln"} \text{ L-infinity Metric (Chebyshev Distance): } \max  p - n $ Defaults to "l2".

## Details

The algorithm implements a hierarchical agglomerative approach where bins are iteratively merged based on minimum pairwise divergence until the `max_bins` constraint is satisfied or convergence is achieved.

### Algorithm Workflow:

1. Input validation and frequency computation
2. Pre-binning of rare categories (if unique categories > `max_n_rebins`)
3. Initialization of pairwise divergence matrix
4. Iterative merging of most similar bin pairs
5. Splitting of heterogeneous bins (if bins < `min_bins`)
6. Final metric computation and WoE-based sorting

**Divergence Measure Selection:** The choice of divergence measure affects the binning structure:

- Information-theoretic measures ("kl", "js", "klj"): Emphasize distributional differences; sensitive to rare events
- Metric measures ("l1", "l2", "ln"): Provide geometric interpretation; robust to outliers
- Chi-square family ("sc", "tr"): Balance between information content and robustness
- Hellinger distance ("he"): Bounded measure; suitable for probability distributions

**Pre-binning Strategy:** When the number of unique categories exceeds `max_n_prebins`, categories with fewer than 5 observations are aggregated into a special "PREBIN\_OTHER" bin to control computational complexity.

### Value

A list containing the binning results with the following components:

`id` Integer vector of bin identifiers (1-indexed)  
`bin` Character vector of bin labels (merged category names)  
`woe` Numeric vector of Weight of Evidence values per bin  
`divergence` Numeric vector of divergence contribution per bin  
`count` Integer vector of total observations per bin  
`count_pos` Integer vector of positive cases (target=1) per bin  
`count_neg` Integer vector of negative cases (target=0) per bin  
`converged` Logical indicating algorithm convergence  
`iterations` Integer count of merge operations performed  
`total_divergence` Numeric total divergence of the binning solution  
`bin_method` Character string of WoE method used  
`divergence_method` Character string of divergence measure used

### References

Zeng, G. (2013). Metric Divergence Measures and Information Value in Credit Scoring. *Journal of Mathematics*, 2013, Article ID 848271. [doi:10.1155/2013/848271](https://doi.org/10.1155/2013/848271)

Kullback, S., & Leibler, R. A. (1951). On Information and Sufficiency. *The Annals of Mathematical Statistics*, 22(1), 79-86.

Lin, J. (1991). Divergence Measures Based on the Shannon Entropy. *IEEE Transactions on Information Theory*, 37(1), 145-151.

### See Also

[ob\\_categorical\\_cm](#) for ChiMerge-based categorical binning

## Examples

```

# Example 1: Basic usage with synthetic credit data
set.seed(42)
n <- 1000

# Simulate occupation categories with varying default rates
occupations <- c(
  "Engineer", "Doctor", "Teacher", "Sales",
  "Manager", "Clerk", "Other"
)
default_probs <- c(0.05, 0.03, 0.08, 0.15, 0.07, 0.12, 0.20)

feature <- sample(occupations, n,
  replace = TRUE,
  prob = c(0.15, 0.10, 0.20, 0.18, 0.12, 0.15, 0.10)
)
target <- sapply(feature, function(x) {
  rbinom(1, 1, default_probs[which(occupations == x)])
})

# Apply optimal binning with L2 divergence
result <- ob_categorical_dmiv(feature, target,
  min_bins = 2,
  max_bins = 4,
  divergence_method = "l2"
)

# Examine binning results
print(data.frame(
  bin = result$bin,
  woe = round(result$woe, 3),
  count = result$count,
  event_rate = round(result$count_pos / result$count, 3)
))

# Example 2: Comparing divergence methods
result_js <- ob_categorical_dmiv(feature, target,
  divergence_method = "js",
  max_bins = 4
)
result_kl <- ob_categorical_dmiv(feature, target,
  divergence_method = "kl",
  max_bins = 4
)

cat("Jensen-Shannon bins:", length(result_js$bin), "\n")
cat("Kullback-Leibler bins:", length(result_kl$bin), "\n")

# Example 3: High cardinality feature with pre-binning
set.seed(123)
postal_codes <- paste0("ZIP_", sprintf("%03d", 1:50))
feature_high_card <- sample(postal_codes, 2000, replace = TRUE)

```

```

target_high_card <- rbinom(2000, 1, 0.1)

result_prebin <- ob_categorical_dmv(
  feature_high_card,
  target_high_card,
  max_n_prebins = 15,
  max_bins = 5
)

cat("Final bins after pre-binning:", length(result_prebin$bin), "\n")
cat("Algorithm converged:", result_prebin$converged, "\n")

```

ob\_categorical\_dp

*Optimal Binning for Categorical Variables using Dynamic Programming*

## Description

Performs supervised discretization of categorical variables using a dynamic programming algorithm with optional monotonicity constraints. This method maximizes the total Information Value (IV) while ensuring optimal bin formation that respects user-defined constraints on bin count and frequency. The algorithm guarantees global optimality through dynamic programming.

## Usage

```

ob_categorical_dp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  bin_separator = "%;%",
  monotonic_trend = "auto"
)

```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Missing values are not permitted.

<code>min_bins</code>	Integer. Minimum number of bins to produce. Must be $\geq 2$ . The algorithm searches for solutions within $[\min\_bins, \max\_bins]$ that maximize total IV. Defaults to 3.
<code>max_bins</code>	Integer. Maximum number of bins to produce. Must be $\geq \min\_bins$ . Defines the upper bound of the search space for the optimal solution. Defaults to 5.
<code>bin_cutoff</code>	Numeric. Minimum proportion of total observations required for a category to remain separate. Categories below this threshold are merged with similar categories. Must be in $(0, 1)$ . Defaults to 0.05.
<code>max_n_prebins</code>	Integer. Maximum number of initial bins before dynamic programming optimization. Controls computational complexity. Must be $\geq 2$ . Defaults to 20.
<code>convergence_threshold</code>	Numeric. Convergence tolerance for the iterative dynamic programming updates. Smaller values require stricter convergence. Must be $> 0$ . Defaults to 1e-6.
<code>max_iterations</code>	Integer. Maximum number of dynamic programming iterations. Prevents excessive computation in edge cases. Must be $> 0$ . Defaults to 1000.
<code>bin_separator</code>	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
<code>monotonic_trend</code>	<p>Character string specifying monotonicity constraint for Weight of Evidence. Must be one of:</p> <ul style="list-style-type: none"> <li>"auto" Automatically determine trend direction (default)</li> <li>"ascending" Enforce increasing WoE across bins</li> <li>"descending" Enforce decreasing WoE across bins</li> <li>"none" No monotonicity constraint</li> </ul> <p>Monotonicity constraints are enforced during the DP optimization phase. Defaults to "auto".</p>

## Details

This implementation uses dynamic programming to find the globally optimal binning solution that maximizes total Information Value subject to constraints.

### Algorithm Workflow:

1. Input validation and data preprocessing
2. Rare category merging (frequencies below `bin_cutoff`)
3. Pre-binning limitation (if categories exceed `max_n_prebins`)
4. Category sorting by event rate
5. Dynamic programming table initialization
6. Iterative DP optimization with optional monotonicity constraints
7. Backtracking to construct optimal bins
8. Final metric computation

### Dynamic Programming Formulation:

Let  $DP[i][k]$  represent the maximum total IV achievable using the first  $i$  categories partitioned into  $k$  bins. The recurrence relation is:

$$DP[i][k] = \max_{j < i} \{DP[j][k - 1] + IV(j + 1, i)\}$$

where  $IV(j + 1, i)$  is the Information Value of a bin containing categories from  $j + 1$  to  $i$ . Monotonicity constraints are enforced by restricting transitions that violate WoE ordering.

### Computational Complexity:

- Time:  $O(n^2 \cdot k \cdot m)$  where  $n$  = categories,  $k$  = max\_bins,  $m$  = iterations
- Space:  $O(n \cdot k)$  for DP tables

### Advantages over Heuristic Methods:

- Guarantees global optimality (within constraint space)
- Explicit monotonicity enforcement
- Deterministic and reproducible results
- Efficient caching mechanism for bin statistics

### Value

A list containing the binning results with the following components:

id Integer vector of bin identifiers (1-indexed)  
 bin Character vector of bin labels (merged category names)  
 woe Numeric vector of Weight of Evidence values per bin  
 iv Numeric vector of Information Value contribution per bin  
 count Integer vector of total observations per bin  
 count\_pos Integer vector of positive cases (target=1) per bin  
 count\_neg Integer vector of negative cases (target=0) per bin  
 event\_rate Numeric vector of event rates per bin  
 total\_iv Numeric total Information Value of the binning solution  
 converged Logical indicating if the DP algorithm converged  
 iterations Integer count of DP iterations performed  
 execution\_time\_ms Numeric execution time in milliseconds

### References

Navas-Palencia, G. (2022). Optimal Binning: Mathematical Programming Formulation. *arXiv preprint arXiv:2001.08025*.

Bellman, R. (1954). The theory of dynamic programming. *Bulletin of the American Mathematical Society*, 60(6), 503-515.

Siddiqi, N. (2017). *Intelligent Credit Scoring: Building and Implementing Better Credit Risk Scorecards* (2nd ed.). Wiley.

Thomas, L. C., Edelman, D. B., & Crook, J. N. (2017). *Credit Scoring and Its Applications* (2nd ed.). SIAM.

**See Also**

[ob\\_categorical\\_cm](#) for ChiMerge-based binning, [ob\\_categorical\\_dmix](#) for divergence measure-based binning

**Examples**

```

# Example 1: Basic usage with monotonic WoE enforcement
set.seed(123)
n_obs <- 1000

# Simulate education levels with increasing default risk
education <- c("High School", "Associate", "Bachelor", "Master", "PhD")
default_probs <- c(0.20, 0.15, 0.10, 0.06, 0.03)

cat_feature <- sample(education, n_obs,
  replace = TRUE,
  prob = c(0.30, 0.25, 0.25, 0.15, 0.05)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, default_probs[which(education == x)])
})

# Apply DP binning with ascending monotonicity
result_dp <- ob_categorical_dp(
  cat_feature,
  bin_target,
  min_bins = 2,
  max_bins = 4,
  monotonic_trend = "ascending"
)

# Display results
print(data.frame(
  Bin = result_dp$bin,
  WoE = round(result_dp$woe, 3),
  IV = round(result_dp$iv, 4),
  Count = result_dp$count,
  EventRate = round(result_dp$event_rate, 3)
))

cat("Total IV:", round(result_dp$total_iv, 4), "\n")
cat("Converged:", result_dp$converged, "\n")

# Example 2: Comparing monotonicity constraints
result_dp_asc <- ob_categorical_dp(
  cat_feature, bin_target,
  max_bins = 3,
  monotonic_trend = "ascending"
)

result_dp_none <- ob_categorical_dp(
  cat_feature, bin_target,

```

```

  max_bins = 3,
  monotonic_trend = "none"
)

cat("\nWith monotonicity:\n")
cat(" Bins:", length(result_dp_asc$bin), "\n")
cat(" Total IV:", round(result_dp_asc$total_iv, 4), "\n")

cat("\nWithout monotonicity:\n")
cat(" Bins:", length(result_dp_none$bin), "\n")
cat(" Total IV:", round(result_dp_none$total_iv, 4), "\n")

# Example 3: High cardinality with pre-binning
set.seed(456)
n_obs_large <- 5000

# Simulate customer segments (high cardinality)
segments <- paste0("Segment_", LETTERS[1:20])
segment_probs <- runif(20, 0.01, 0.20)

cat_feature_hc <- sample(segments, n_obs_large, replace = TRUE)
bin_target_hc <- rbinom(
  n_obs_large, 1,
  segment_probs[match(cat_feature_hc, segments)])
)

result_dp_hc <- ob_categorical_dp(
  cat_feature_hc,
  bin_target_hc,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.03,
  max_n_prebins = 10
)

cat("\nHigh cardinality example:\n")
cat(" Original categories:", length(unique(cat_feature_hc)), "\n")
cat(" Final bins:", length(result_dp_hc$bin), "\n")
cat(" Execution time:", result_dp_hc$execution_time_ms, "ms\n")

# Example 4: Handling missing values
set.seed(789)
cat_feature_na <- cat_feature
cat_feature_na[sample(n_obs, 50)] <- NA # Introduce 5% missing

result_dp_na <- ob_categorical_dp(
  cat_feature_na,
  bin_target,
  min_bins = 2,
  max_bins = 4
)

# Check if NA was treated as a category

```

```

na_bin <- grep("NA", result_dp_na$bin, value = TRUE)
if (length(na_bin) > 0) {
  cat("\nNA handling:\n")
  cat(" Bin containing NA:", na_bin, "\n")
}

```

---

ob\_categorical\_fetb     *Optimal Binning for Categorical Variables using Fisher's Exact Test*

---

## Description

Performs supervised discretization of categorical variables using Fisher's Exact Test as the similarity criterion for hierarchical bin merging. This method iteratively merges the most statistically similar bins (highest p-value) while enforcing Weight of Evidence monotonicity, providing a statistically rigorous approach to optimal binning.

## Usage

```

ob_categorical_fetb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  bin_separator = "%;%"
)

```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Missing values are not permitted.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 2$ . The algorithm will not merge below this threshold. Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm merges bins until this constraint is satisfied. Defaults to 5.
bin_cutoff	Numeric. Minimum proportion of total observations required for a category to avoid being classified as rare. Rare categories are pre-merged before the main algorithm. Must be in (0, 1). Defaults to 0.05.

max_n_prebins	Integer. Maximum number of initial bins before the merging phase. Controls computational complexity for high-cardinality features. Must be $\geq 2$ . Defaults to 20.
convergence_threshold	Numeric. Convergence tolerance based on Information Value change between iterations. Algorithm stops when $ \Delta IV  < \text{convergence\_threshold}$ . Must be $> 0$ . Defaults to 1e-6.
max_iterations	Integer. Maximum number of merge operations allowed. Prevents excessive computation. Must be $> 0$ . Defaults to 1000.
bin_separator	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".

## Details

This algorithm employs Fisher's Exact Test to quantify statistical similarity between bins based on their  $2 \times 2$  contingency tables. Unlike chi-square based methods, Fisher's test provides exact p-values without relying on asymptotic approximations, making it particularly suitable for small sample sizes.

### Algorithm Workflow:

1. Data preprocessing and frequency computation
2. Rare category identification and pre-merging (frequencies  $< \text{bin\_cutoff}$ )
3. Initial bin creation (one category per bin)
4. Iterative merging phase:
  - Compute Fisher's Exact Test p-values for all adjacent bin pairs
  - Merge the pair with the **highest** p-value (most similar)
  - Enforce WoE monotonicity after each merge
  - Check convergence based on IV change
5. Final monotonicity enforcement

### Fisher's Exact Test:

For two bins with contingency table:

	Bin 1	Bin 2
Positives	$a$	$c$
Negatives	$b$	$d$

The exact probability under the null hypothesis of independence is:

$$p = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{n! \cdot a! \cdot b! \cdot c! \cdot d!}$$

where  $n = a + b + c + d$ . Higher p-values indicate greater similarity (less evidence against the null hypothesis of identical distributions).

### Key Features:

- **Exact inference:** No asymptotic approximations required
- **Small sample robustness:** Valid for any sample size
- **Automatic monotonicity:** WoE ordering enforced after each merge
- **Efficient caching:** Log-factorial and p-value caching for speed
- **Rare category handling:** Pre-merging prevents sparse bins

### Computational Complexity:

- Time:  $O(k^2 \cdot m)$  where  $k$  = initial bins,  $m$  = iterations
- Space:  $O(k + n_{max})$  for bins and factorial cache

### Value

A list containing the binning results with the following components:

`id` Integer vector of bin identifiers (1-indexed)  
`bin` Character vector of bin labels (merged category names)  
`woe` Numeric vector of Weight of Evidence values per bin  
`iv` Numeric vector of Information Value contribution per bin  
`count` Integer vector of total observations per bin  
`count_pos` Integer vector of positive cases (target=1) per bin  
`count_neg` Integer vector of negative cases (target=0) per bin  
`converged` Logical indicating algorithm convergence  
`iterations` Integer count of merge operations performed

### References

Fisher, R. A. (1922). On the interpretation of chi-squared from contingency tables, and the calculation of P. *Journal of the Royal Statistical Society*, 85(1), 87-94. [doi:10.2307/2340521](https://doi.org/10.2307/2340521)

Agresti, A. (2013). *Categorical Data Analysis* (3rd ed.). Wiley.

Mehta, C. R., & Patel, N. R. (1983). A network algorithm for performing Fisher's exact test in rxc contingency tables. *Journal of the American Statistical Association*, 78(382), 427-434.

Zeng, G. (2014). A necessary condition for a good binning algorithm in credit scoring. *Applied Mathematical Sciences*, 8(65), 3229-3242.

### See Also

`ob_categorical_cm` for ChiMerge-based binning, `ob_categorical_dp` for dynamic programming approach, `ob_categorical_dmv` for divergence measure-based binning

## Examples

```

# Example 1: Basic usage with Fisher's Exact Test
set.seed(42)
n_obs <- 800

# Simulate customer segments with different risk profiles
segments <- c("Premium", "Standard", "Basic", "Budget", "Economy")
risk_rates <- c(0.05, 0.10, 0.15, 0.22, 0.30)

cat_feature <- sample(segments, n_obs,
  replace = TRUE,
  prob = c(0.15, 0.25, 0.30, 0.20, 0.10)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, risk_rates[which(segments == x)])
})

# Apply Fisher's Exact Test binning
result_fetb <- ob_categorical_fetb(
  cat_feature,
  bin_target,
  min_bins = 2,
  max_bins = 4
)

# Display results
print(data.frame(
  Bin = result_fetb$bin,
  WoE = round(result_fetb$woe, 3),
  IV = round(result_fetb$iv, 4),
  Count = result_fetb$count,
  EventRate = round(result_fetb$count_pos / result_fetb$count, 3)
))

cat("\nAlgorithm converged:", result_fetb$converged, "\n")
cat("Iterations performed:", result_fetb$iterations, "\n")

# Example 2: Comparing with ChiMerge method
result_cm <- ob_categorical_cm(
  cat_feature,
  bin_target,
  min_bins = 2,
  max_bins = 4
)

cat("\nFisher's Exact Test:\n")
cat("  Final bins:", length(result_fetb$bin), "\n")
cat("  Total IV:", round(sum(result_fetb$iv), 4), "\n")

cat("\nChiMerge:\n")
cat("  Final bins:", length(result_cm$bin), "\n")
cat("  Total IV:", round(sum(result_cm$iv), 4), "\n")

```

```

# Example 3: Small sample size (Fisher's advantage)
set.seed(123)
n_obs_small <- 150

# Small sample with sparse categories
occupation <- c(
  "Doctor", "Lawyer", "Teacher", "Engineer",
  "Sales", "Manager"
)

cat_feature_small <- sample(occupation, n_obs_small,
  replace = TRUE,
  prob = c(0.10, 0.10, 0.20, 0.25, 0.20, 0.15)
)
bin_target_small <- rbinom(n_obs_small, 1, 0.12)

result_fetb_small <- ob_categorical_fetb(
  cat_feature_small,
  bin_target_small,
  min_bins = 2,
  max_bins = 3,
  bin_cutoff = 0.03 # Allow smaller bins for small sample
)

cat("\nSmall sample binning:\n")
cat(" Observations:", n_obs_small, "\n")
cat(" Original categories:", length(unique(cat_feature_small)), "\n")
cat(" Final bins:", length(result_fetb_small$bin), "\n")
cat(" Converged:", result_fetb_small$converged, "\n")

# Example 4: High cardinality with rare categories
set.seed(789)
n_obs_hc <- 2000

# Simulate product codes (high cardinality)
product_codes <- paste0("PROD_", sprintf("%03d", 1:30))

cat_feature_hc <- sample(product_codes, n_obs_hc,
  replace = TRUE,
  prob = c(
    rep(0.05, 10), rep(0.02, 10),
    rep(0.01, 10)
  )
)
bin_target_hc <- rbinom(n_obs_hc, 1, 0.08)

result_fetb_hc <- ob_categorical_fetb(
  cat_feature_hc,
  bin_target_hc,
  min_bins = 3,
  max_bins = 6,
  bin_cutoff = 0.02,
)

```

```

max_n_prebins = 15
)

cat("\nHigh cardinality example:\n")
cat(" Original categories:", length(unique(cat_feature_hc)), "\n")
cat(" Final bins:", length(result_fetb_hc$bin), "\n")
cat(" Iterations:", result_fetb_hc$iterations, "\n")

# Check for rare category merging
for (i in seq_along(result_fetb_hc$bin)) {
  n_merged <- length(strsplit(result_fetb_hc$bin[i], "%;%")[[1]])
  if (n_merged > 1) {
    cat(" Bin", i, "contains", n_merged, "merged categories\n")
  }
}

# Example 5: Missing value handling
set.seed(456)
cat_feature_na <- cat_feature
na_indices <- sample(n_obs, 40) # 5% missing
cat_feature_na[na_indices] <- NA

result_fetb_na <- ob_categorical_fetb(
  cat_feature_na,
  bin_target,
  min_bins = 2,
  max_bins = 4
)

# Check NA treatment
na_bin_idx <- grep("NA", result_fetb_na$bin)
if (length(na_bin_idx) > 0) {
  cat("\nMissing value handling:\n")
  cat(" NA bin:", result_fetb_na$bin[na_bin_idx], "\n")
  cat(" NA count:", result_fetb_na$count[na_bin_idx], "\n")
  cat(" NA WoE:", round(result_fetb_na$woe[na_bin_idx], 3), "\n")
}

```

---

## Description

Performs supervised discretization of categorical variables using a greedy bottom-up merging strategy that iteratively combines bins to maximize total Information Value (IV). This approach uses Bayesian smoothing for numerical stability and employs adaptive monotonicity constraints, providing a fast approximation to optimal binning suitable for high-cardinality features.

**Usage**

```
ob_categorical_gmb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

**Arguments**

<code>feature</code>	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
<code>target</code>	An integer vector of binary outcomes (0/1) corresponding to each observation in <code>feature</code> . Missing values are not permitted.
<code>min_bins</code>	Integer. Minimum number of bins to produce. Must be $\geq 2$ . Merging stops when this threshold is reached. Defaults to 3.
<code>max_bins</code>	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm terminates when bins are reduced to this number. Defaults to 5.
<code>bin_cutoff</code>	Numeric. Minimum proportion of total observations required for a category to remain separate during initialization. Categories below this threshold are pre-merged. Must be in (0, 1). Defaults to 0.05.
<code>max_n_prebins</code>	Integer. Maximum number of initial bins before the greedy merging phase. Controls computational complexity. Must be $\geq \text{min\_bins}$ . Defaults to 20.
<code>bin_separator</code>	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
<code>convergence_threshold</code>	Numeric. Convergence tolerance for IV change between iterations. Algorithm stops when $ \Delta IV  < \text{convergence\_threshold}$ . Must be $> 0$ . Defaults to 1e-6.
<code>max_iterations</code>	Integer. Maximum number of merge operations allowed. Prevents excessive computation. Must be $> 0$ . Defaults to 1000.

**Details**

The Greedy Merge Binning (GMB) algorithm employs a bottom-up approach where bins are iteratively merged based on maximum IV improvement. Unlike exact optimization methods (e.g., dynamic programming), GMB provides approximate solutions with significantly reduced computational cost.

**Algorithm Workflow:**

1. Input validation and preprocessing
2. Initial bin creation (one category per bin)

3. Rare category merging (frequencies < bin\_cutoff)
4. Pre-bin limitation (if bins > max\_n\_prebins)
5. Greedy merging phase:
  - Evaluate IV for all possible adjacent bin merges
  - Select merge that maximizes total IV
  - Apply tie-breaking rules for similar merges
  - Update IV cache incrementally
  - Check convergence criteria
6. Adaptive monotonicity enforcement
7. Final metric computation

### Bayesian Smoothing:

To prevent numerical instability with sparse bins, WoE is calculated using Bayesian smoothing:

$$WoE_i = \ln \left( \frac{p_i + \alpha_p}{n_i + \alpha_n} \right)$$

where  $\alpha_p$  and  $\alpha_n$  are prior pseudocounts proportional to the overall event rate. This regularization ensures stable WoE estimates even for bins with zero events.

### Greedy Selection Criterion:

At each iteration, the algorithm evaluates the IV gain for merging adjacent bins  $i$  and  $j$ :

$$\Delta IV_{i,j} = IV_{merged}(i,j) - (IV_i + IV_j)$$

The pair with maximum  $\Delta IV$  is merged. Early stopping occurs if  $\Delta IV > 0.05 \cdot IV_{current}$  (5% improvement threshold).

### Tie Handling:

When multiple merges yield similar IV gains (within 10x convergence threshold), the algorithm prefers merges that produce more balanced bins, breaking ties based on size difference:

$$balance\_score = |count_i - count_j|$$

### Computational Complexity:

- Time:  $O(k^2 \cdot m)$  where  $k = \text{bins}$ ,  $m = \text{iterations}$
- Space:  $O(k^2)$  for IV cache (optional)
- Typical runtime: 10-100x faster than exact methods for  $k > 20$

### Advantages:

- Fast execution for high-cardinality features
- Incremental IV caching for efficiency
- Bayesian smoothing prevents overfitting
- Adaptive monotonicity with gradient relaxation

- Handles imbalanced datasets robustly

#### Limitations:

- Approximate solution (not guaranteed global optimum)
- Greedy nature may miss better non-adjacent merges
- Sensitive to initialization order

#### Value

A list containing the binning results with the following components:

`id` Integer vector of bin identifiers (1-indexed)  
`bin` Character vector of bin labels (merged category names)  
`woe` Numeric vector of Weight of Evidence values per bin  
`iv` Numeric vector of Information Value contribution per bin  
`count` Integer vector of total observations per bin  
`count_pos` Integer vector of positive cases (target=1) per bin  
`count_neg` Integer vector of negative cases (target=0) per bin  
`total_iv` Numeric total Information Value of the binning solution  
`converged` Logical indicating algorithm convergence  
`iterations` Integer count of merge operations performed

#### References

Navas-Palencia, G. (2020). Optimal binning: mathematical programming formulation and solution approach. *Expert Systems with Applications*, 158, 113508. [doi:10.1016/j.eswa.2020.113508](https://doi.org/10.1016/j.eswa.2020.113508)

Good, I. J. (1965). The Estimation of Probabilities: An Essay on Modern Bayesian Methods. MIT Press.

Zeng, G. (2014). A necessary condition for a good binning algorithm in credit scoring. *Applied Mathematical Sciences*, 8(65), 3229-3242.

Mironchyk, P., & Tchistiakov, V. (2017). Monotone optimal binning algorithm for credit risk modeling. *SSRN Electronic Journal*. [doi:10.2139/ssrn.2978774](https://doi.org/10.2139/ssrn.2978774)

#### See Also

[ob\\_categorical\\_dp](#) for exact optimization via dynamic programming, [ob\\_categorical\\_cm](#) for ChiMerge-based binning, [ob\\_categorical\\_fetb](#) for Fisher's Exact Test binning

#### Examples

```

# Example 1: Basic greedy merge binning
set.seed(123)
n_obs <- 1500

# Simulate customer types with varying risk
customer_types <- c(

```

```
"Premium", "Gold", "Silver", "Bronze",
"Basic", "Trial"
)
risk_probs <- c(0.02, 0.05, 0.10, 0.15, 0.22, 0.35)

cat_feature <- sample(customer_types, n_obs,
  replace = TRUE,
  prob = c(0.10, 0.15, 0.25, 0.25, 0.15, 0.10)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, risk_probs[which(customer_types == x)])
})

# Apply greedy merge binning
result_gmb <- ob_categorical_gmb(
  cat_feature,
  bin_target,
  min_bins = 3,
  max_bins = 4
)

# Display results
print(data.frame(
  Bin = result_gmb$bin,
  WoE = round(result_gmb$woe, 3),
  IV = round(result_gmb$iv, 4),
  Count = result_gmb$count,
  EventRate = round(result_gmb$count_pos / result_gmb$count, 3)
))

cat("\nTotal IV:", round(result_gmb$total_iv, 4), "\n")
cat("Converged:", result_gmb$converged, "\n")
cat("Iterations:", result_gmb$iterations, "\n")

# Example 2: Comparing speed with exact methods
set.seed(456)
n_obs <- 3000

# High cardinality feature
regions <- paste0("Region_", sprintf("%02d", 1:25))
cat_feature_hc <- sample(regions, n_obs, replace = TRUE)
bin_target_hc <- rbinom(n_obs, 1, 0.12)

# Greedy approach (fast)
time_gmb <- system.time({
  result_gmb_hc <- ob_categorical_gmb(
    cat_feature_hc,
    bin_target_hc,
    min_bins = 3,
    max_bins = 6,
    max_n_prebins = 20
  )
})
```

```

# Dynamic programming (exact but slower)
time_dp <- system.time({
  result_dp_hc <- ob_categorical_dp(
    cat_feature_hc,
    bin_target_hc,
    min_bins = 3,
    max_bins = 6,
    max_n_prebins = 20
  )
})

cat("\nPerformance comparison (high cardinality):\n")
cat("  GMB time:", round(time_gmb[3], 3), "seconds\n")
cat("  DP time:", round(time_dp[3], 3), "seconds\n")
cat("  Speedup:", round(time_dp[3] / time_gmb[3], 1), "x\n")
cat("\n  GMB IV:", round(result_gmb_hc$total_iv, 4), "\n")
cat("  DP IV:", round(result_dp_hc$total_iv, 4), "\n")

# Example 3: Convergence behavior
set.seed(789)
n_obs_conv <- 1000

# Feature with natural groupings
education <- c("PhD", "Master", "Bachelor", "HighSchool", "NoHighSchool")
cat_feature_conv <- sample(education, n_obs_conv,
  replace = TRUE,
  prob = c(0.05, 0.15, 0.35, 0.30, 0.15)
)
bin_target_conv <- sapply(cat_feature_conv, function(x) {
  probs <- c(0.02, 0.05, 0.08, 0.15, 0.25)
  rbinom(1, 1, probs[which(education == x)])
})

# Test different convergence thresholds
thresholds <- c(1e-3, 1e-6, 1e-9)

for (thresh in thresholds) {
  result_conv <- ob_categorical_gmb(
    cat_feature_conv,
    bin_target_conv,
    min_bins = 2,
    max_bins = 4,
    convergence_threshold = thresh
  )

  cat(sprintf(
    "\nThreshold %.0e: %d iterations, converged=%s\n",
    thresh, result_conv$iterations, result_conv$converged
  ))
}

# Example 4: Handling rare categories

```

```
set.seed(321)
n_obs_rare <- 2000

# Simulate with many rare categories
products <- c(paste0("Common_", 1:5), paste0("Rare_", 1:15))
product_probs <- c(rep(0.15, 5), rep(0.01, 15))

cat_feature_rare <- sample(products, n_obs_rare,
  replace = TRUE,
  prob = product_probs
)
bin_target_rare <- rbinom(n_obs_rare, 1, 0.10)

result_gmb_rare <- ob_categorical_gmb(
  cat_feature_rare,
  bin_target_rare,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.03 # Aggressive rare category merging
)

cat("\nRare category handling:\n")
cat(" Original categories:", length(unique(cat_feature_rare)), "\n")
cat(" Final bins:", length(result_gmb_rare$bin), "\n")

# Count merged rare categories
for (i in seq_along(result_gmb_rare$bin)) {
  n_merged <- length(strsplit(result_gmb_rare$bin[i], "%;%")[[1]])
  if (n_merged > 1) {
    cat(sprintf(" Bin %d: %d categories merged\n", i, n_merged))
  }
}

# Example 5: Imbalanced dataset robustness
set.seed(555)
n_obs_imb <- 1200

# Highly imbalanced target (2% event rate)
cat_feature_imb <- sample(c("A", "B", "C", "D", "E"),
  n_obs_imb,
  replace = TRUE
)
bin_target_imb <- rbinom(n_obs_imb, 1, 0.02)

result_gmb_imb <- ob_categorical_gmb(
  cat_feature_imb,
  bin_target_imb,
  min_bins = 2,
  max_bins = 3
)

cat("\nImbalanced dataset:\n")
cat(" Event rate:", round(mean(bin_target_imb), 4), "\n")
```

```

cat(" Total events:", sum(bin_target_imb), "\n")
cat(" Bins created:", length(result_gmb_imb$bin), "\n")
cat(" WoE range:", sprintf(
  "[%.2f, %.2f]\n",
  min(result_gmb_imb$woe),
  max(result_gmb_imb$woe)
))

```

---

<b>ob_categorical_ivb</b>	<i>Optimal Binning for Categorical Variables using Information Value Dynamic Programming</i>
---------------------------	--

---

## Description

Performs supervised discretization of categorical variables using a dynamic programming algorithm specifically designed to maximize total Information Value (IV). This implementation employs Bayesian smoothing for numerical stability, maintains monotonic Weight of Evidence constraints, and uses efficient caching strategies for optimal performance with high-cardinality features.

## Usage

```

ob_categorical_ivb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

<b>feature</b>	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
<b>target</b>	An integer vector of binary outcomes (0/1) corresponding to each observation in <b>feature</b> . Missing values are not permitted.
<b>min_bins</b>	Integer. Minimum number of bins to produce. Must be $\geq 2$ . The algorithm searches for solutions within $[\min\_bins, \max\_bins]$ that maximize total IV. Defaults to 3.
<b>max_bins</b>	Integer. Maximum number of bins to produce. Must be $\geq \min\_bins$ . Defines the upper bound of the search space. Defaults to 5.

bin_cutoff	Numeric. Minimum proportion of total observations required for a category to remain separate. Categories below this threshold are pre-merged before the optimization phase. Must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. Maximum number of initial bins before dynamic programming optimization. Controls computational complexity for high-cardinality features. Must be $\geq 2$ . Defaults to 20.
bin_separator	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
convergence_threshold	Numeric. Convergence tolerance for the iterative optimization process based on IV change. Algorithm stops when $ \Delta IV  < \text{convergence\_threshold}$ . Must be $> 0$ . Defaults to 1e-6.
max_iterations	Integer. Maximum number of optimization iterations. Prevents excessive computation. Must be $> 0$ . Defaults to 1000.

## Details

The Information Value Binning (IVB) algorithm uses dynamic programming to find the globally optimal binning solution that maximizes total IV subject to constraints on bin count and monotonicity.

### Algorithm Workflow:

1. Input validation and preprocessing
2. Single-pass category counting and statistics computation
3. Rare category pre-merging (frequencies  $< \text{bin\_cutoff}$ )
4. Pre-bin limitation (if categories  $> \text{max\_n\_prebins}$ )
5. Category sorting by event rate
6. Cumulative statistics cache initialization
7. Dynamic programming table computation:
  - State:  $DP[i][k] = \max$  IV using first  $i$  categories in  $k$  bins
  - Transition:  $DP[i][k] = \max_j \{DP[j][k - 1] + IV(j + 1, i)\}$
  - Banded optimization to skip infeasible splits
8. Backtracking to reconstruct optimal bins
9. Adaptive monotonicity enforcement
10. Final metric computation with Bayesian smoothing

### Dynamic Programming Formulation:

Let  $DP[i][k]$  represent the maximum total IV achievable using the first  $i$  categories (sorted by event rate) partitioned into  $k$  bins.

### Recurrence relation:

$$DP[i][k] = \max_{k-1 \leq j < i} \{DP[j][k - 1] + IV(j + 1, i)\}$$

**Base case:**

$$DP[i][1] = IV(1, i) \quad \forall i$$

where  $IV(j+1, i)$  is the Information Value of a bin containing categories from  $j+1$  to  $i$ .

**Bayesian Smoothing:**

To prevent numerical instability and overfitting with sparse bins, WoE and IV are calculated using Bayesian smoothing with pseudocounts:

$$p'_i = \frac{n_{i, \text{pos}} + \alpha_p}{N_{\text{pos}} + \alpha_{\text{total}}}$$

$$n'_i = \frac{n_{i, \text{neg}} + \alpha_n}{N_{\text{neg}} + \alpha_{\text{total}}}$$

where  $\alpha_p$  and  $\alpha_n$  are prior pseudocounts proportional to the overall event rate, and  $\alpha_{\text{total}} = 1.0$  (prior strength).

$$WoE_i = \ln \left( \frac{p'_i}{n'_i} \right)$$

$$IV_i = (p'_i - n'_i) \times WoE_i$$

**Adaptive Monotonicity Enforcement:**

After finding the optimal bins, the algorithm enforces WoE monotonicity by:

1. Computing average WoE gap:  $\bar{\Delta} = \frac{1}{k-1} \sum_{i=1}^{k-1} |WoE_{i+1} - WoE_i|$
2. Setting adaptive threshold:  $\tau = \min(\epsilon, 0.01\bar{\Delta})$
3. Identifying worst violation:  $i^* = \arg \max_i \{WoE_i - WoE_{i+1}\}$
4. Evaluating forward and backward merges by IV retention
5. Selecting merge direction that maximizes total IV

**Computational Complexity:**

- Time:  $O(k^2 \cdot n)$  where  $k = \text{max\_bins}$ ,  $n = \text{categories}$
- Space:  $O(k \cdot n)$  for DP tables and cumulative caches
- IV calculations are  $O(1)$  due to cumulative statistics caching

**Advantages over Alternative Methods:**

- **Global optimality:** Guaranteed maximum IV (within constraint space)
- **Bayesian regularization:** Robust to sparse bins and class imbalance
- **Efficient caching:** Cumulative stats and IV memoization
- **Banded optimization:** Reduced search space via feasibility pruning
- **Adaptive monotonicity:** Context-aware threshold for enforcement

**Comparison with Related Methods:**

- **vs DP (general):** IVB specifically optimizes IV; general DP more flexible
- **vs GMB:** IVB guarantees optimality; GMB is faster but approximate
- **vs ChiMerge:** IVB uses IV criterion; ChiMerge uses chi-square

## Value

A list containing the binning results with the following components:

- id Integer vector of bin identifiers (1-indexed)
- bin Character vector of bin labels (merged category names)
- woe Numeric vector of Weight of Evidence values per bin
- iv Numeric vector of Information Value contribution per bin
- count Integer vector of total observations per bin
- count\_pos Integer vector of positive cases (target=1) per bin
- count\_neg Integer vector of negative cases (target=0) per bin
- total\_iv Numeric total Information Value of the binning solution
- converged Logical indicating algorithm convergence
- iterations Integer count of optimization iterations performed

## References

Navas-Palencia, G. (2020). Optimal binning: mathematical programming formulation and solution approach. *Expert Systems with Applications*, 158, 113508. [doi:10.1016/j.eswa.2020.113508](https://doi.org/10.1016/j.eswa.2020.113508)

Bellman, R. (1957). *Dynamic Programming*. Princeton University Press.

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Good, I. J. (1965). *The Estimation of Probabilities: An Essay on Modern Bayesian Methods*. MIT Press.

Anderson, R. (2007). *The Credit Scoring Toolkit: Theory and Practice for Retail Credit Risk Management and Decision Automation*. Oxford University Press.

## See Also

[ob\\_categorical\\_dp](#) for general dynamic programming binning, [ob\\_categorical\\_gmb](#) for greedy merge approximation, [ob\\_categorical\\_cm](#) for ChiMerge-based binning

## Examples

```
# Example 1: Basic IV optimization with Bayesian smoothing
set.seed(42)
n_obs <- 1200

# Simulate industry sectors with varying default risk
industries <- c(
  "Technology", "Healthcare", "Finance", "Manufacturing",
  "Retail", "Energy"
)
default_rates <- c(0.03, 0.05, 0.08, 0.12, 0.18, 0.25)

cat_feature <- sample(industries, n_obs,
  replace = TRUE,
```

```

prob = c(0.20, 0.18, 0.22, 0.18, 0.12, 0.10)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, default_rates[which(industries == x)])
})

# Apply IVB optimization
result_ivb <- ob_categorical_ivb(
  cat_feature,
  bin_target,
  min_bins = 3,
  max_bins = 4
)

# Display results
print(data.frame(
  Bin = result_ivb$bin,
  WoE = round(result_ivb$woe, 3),
  IV = round(result_ivb$iv, 4),
  Count = result_ivb$count,
  EventRate = round(result_ivb$count_pos / result_ivb$count, 3)
))

cat("\nTotal IV (maximized):", round(result_ivb$total_iv, 4), "\n")
cat("Converged:", result_ivb$converged, "\n")
cat("Iterations:", result_ivb$iterations, "\n")

# Example 2: Comparing IV optimization with other methods
set.seed(123)
n_obs_comp <- 1500

regions <- c("North", "South", "East", "West", "Central")
cat_feature_comp <- sample(regions, n_obs_comp, replace = TRUE)
bin_target_comp <- rbinom(n_obs_comp, 1, 0.15)

# IVB (IV-optimized)
result_ivb_comp <- ob_categorical_ivb(
  cat_feature_comp, bin_target_comp,
  min_bins = 2, max_bins = 3
)

# GMB (greedy approximation)
result_gmb_comp <- ob_categorical_gmb(
  cat_feature_comp, bin_target_comp,
  min_bins = 2, max_bins = 3
)

# DP (general optimization)
result_dp_comp <- ob_categorical_dp(
  cat_feature_comp, bin_target_comp,
  min_bins = 2, max_bins = 3
)

```

```

cat("\nMethod comparison:\n")
cat(" IVB total IV:", round(result_ivb_comp$total_iv, 4), "\n")
cat(" GMB total IV:", round(result_gmb_comp$total_iv, 4), "\n")
cat(" DP total IV:", round(result_dp_comp$total_iv, 4), "\n")
cat("\nIVB typically achieves highest IV due to explicit optimization\n")

```

---

ob\_categorical\_jedi    *Optimal Binning for Categorical Variables using JEDI Algorithm*

---

## Description

Performs supervised discretization of categorical variables using the Joint Entropy-Driven Information Maximization (JEDI) algorithm. This advanced method combines information-theoretic optimization with intelligent bin merging strategies, employing Bayesian smoothing for numerical stability and adaptive monotonicity enforcement to produce robust, interpretable binning solutions.

## Usage

```

ob_categorical_jedi(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Missing values are not permitted.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 2$ . The algorithm will not merge below this threshold. Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm iteratively merges until this constraint is satisfied. Defaults to 5.
bin_cutoff	Numeric. Minimum proportion of total observations required for a category to remain separate during initialization. Categories below this threshold are pre-merged into an "Others" bin. Must be in (0, 1). Defaults to 0.05.

max_n_prebins	Integer. Maximum number of initial bins before the main optimization phase. Controls computational complexity for high-cardinality features. Must be $\geq$ min_bins. Defaults to 20.
bin_separator	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
convergence_threshold	Numeric. Convergence tolerance based on Information Value change between iterations. Algorithm stops when $ \Delta IV  < \text{convergence\_threshold}$ . Must be $> 0$ . Defaults to 1e-6.
max_iterations	Integer. Maximum number of optimization iterations. Prevents infinite loops in edge cases. Must be $> 0$ . Defaults to 1000.

## Details

The JEDI (Joint Entropy-Driven Information Maximization) algorithm represents a sophisticated approach to categorical binning that jointly optimizes Information Value while maintaining monotonic Weight of Evidence constraints through intelligent violation detection and repair strategies.

### Algorithm Workflow:

1. Input validation and preprocessing
2. Initial bin creation (one category per bin)
3. Rare category merging (frequencies  $< \text{bin\_cutoff}$ )
4. WoE-based monotonic sorting
5. Pre-bin limitation via minimal IV-loss merging
6. Main optimization loop:
  - Monotonicity violation detection (peaks and valleys)
  - Violation severity quantification
  - Intelligent merge selection (minimize IV loss)
  - Convergence monitoring
  - Best solution tracking
7. Final constraint satisfaction (max\_bins enforcement)
8. Bayesian-smoothed metric computation

### Joint Entropy-Driven Optimization:

Unlike greedy algorithms that optimize locally, JEDI considers the global impact of each merge on total Information Value:

$$IV_{total} = \sum_{i=1}^k (p_i - n_i) \times \ln \left( \frac{p_i}{n_i} \right)$$

For each potential merge of bins  $j$  and  $j + 1$ , JEDI evaluates:

$$\Delta IV_{j,j+1} = IV_{current} - IV_{merged}(j, j + 1)$$

The pair with minimum  $\Delta IV$  (least information loss) is selected.

### Violation Detection and Repair:

JEDI identifies two types of monotonicity violations:

- **Peaks:**  $WoE_i > WoE_{i-1}$  and  $WoE_i > WoE_{i+1}$
- **Valleys:**  $WoE_i < WoE_{i-1}$  and  $WoE_i < WoE_{i+1}$

For each violation, severity is quantified as:

$$severity_i = \max\{|WoE_i - WoE_{i-1}|, |WoE_i - WoE_{i+1}|\}$$

The algorithm prioritizes fixing the most severe violation first, evaluating both forward merge  $(i, i+1)$  and backward merge  $(i-1, i)$  to select the option that minimizes information loss.

### Bayesian Smoothing:

To ensure numerical stability with sparse bins, JEDI applies Bayesian smoothing:

$$p'_i = \frac{n_{i, \text{pos}} + \alpha_p}{N_{\text{pos}} + \alpha_{\text{total}}}$$

$$n'_i = \frac{n_{i, \text{neg}} + \alpha_n}{N_{\text{neg}} + \alpha_{\text{total}}}$$

where prior pseudocounts are proportional to overall prevalence:

$$\alpha_p = \alpha_{\text{total}} \times \frac{N_{\text{pos}}}{N_{\text{pos}} + N_{\text{neg}}}$$

$$\alpha_n = \alpha_{\text{total}} - \alpha_p$$

with  $\alpha_{\text{total}} = 1.0$  as the prior strength parameter.

### Adaptive Monotonicity Threshold:

Rather than using a fixed threshold, JEDI computes a context-aware tolerance:

$$\bar{\Delta} = \frac{1}{k-1} \sum_{i=1}^{k-1} |WoE_{i+1} - WoE_i|$$

$$\tau = \min(\epsilon, 0.01\bar{\Delta})$$

This adaptive approach prevents over-merging when natural WoE gaps are small.

### Computational Complexity:

- Time:  $O(k^2 \cdot m)$  where  $k$  = bins,  $m$  = iterations
- Space:  $O(k^2)$  for IV cache
- Cache hit rate typically  $> 70\%$  for  $k > 10$

### Key Innovations:

- **Joint optimization:** Global IV consideration (vs. local greedy)
- **Smart violation repair:** Severity-based prioritization

- **Bidirectional merge evaluation:** Forward vs. backward analysis
- **Best solution tracking:** Retains optimal intermediate states
- **Adaptive thresholds:** Context-aware monotonicity tolerance

#### Comparison with Related Methods:

Method	Optimization	Monotonicity	Speed
JEDI	Joint/Global	Adaptive	Medium
IVB	DP (Exact)	Enforced	Slow
GMB	Greedy/Local	Enforced	Fast
ChiMerge	Statistical	Optional	Fast

#### Value

A list containing the binning results with the following components:

id Integer vector of bin identifiers (1-indexed)  
 bin Character vector of bin labels (merged category names)  
 woe Numeric vector of Weight of Evidence values per bin  
 iv Numeric vector of Information Value contribution per bin  
 count Integer vector of total observations per bin  
 count\_pos Integer vector of positive cases (target=1) per bin  
 count\_neg Integer vector of negative cases (target=0) per bin  
 total\_iv Numeric total Information Value of the binning solution  
 converged Logical indicating algorithm convergence  
 iterations Integer count of optimization iterations performed

#### References

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Kullback, S. (1959). *Information Theory and Statistics*. Wiley.

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Zeng, G. (2014). A necessary condition for a good binning algorithm in credit scoring. *Applied Mathematical Sciences*, 8(65), 3229-3242.

#### See Also

[ob\\_categorical\\_ivb](#) for Information Value DP optimization, [ob\\_categorical\\_gmb](#) for greedy merge binning, [ob\\_categorical\\_dp](#) for general dynamic programming, [ob\\_categorical\\_cm](#) for ChiMerge-based binning

## Examples

```

# Example 1: Basic JEDI optimization
set.seed(42)
n_obs <- 1500

# Simulate employment types with risk gradient
employment <- c(
  "Permanent", "Contract", "Temporary", "SelfEmployed",
  "Unemployed", "Student", "Retired"
)
risk_rates <- c(0.03, 0.08, 0.15, 0.12, 0.35, 0.25, 0.10)

cat_feature <- sample(employment, n_obs,
  replace = TRUE,
  prob = c(0.35, 0.20, 0.15, 0.12, 0.08, 0.06, 0.04)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, risk_rates[which(employment == x)])
})

# Apply JEDI algorithm
result_jedi <- ob_categorical_jedi(
  cat_feature,
  bin_target,
  min_bins = 3,
  max_bins = 5
)

# Display results
print(data.frame(
  Bin = result_jedi$bin,
  WoE = round(result_jedi$woe, 3),
  IV = round(result_jedi$iv, 4),
  Count = result_jedi$count,
  EventRate = round(result_jedi$count_pos / result_jedi$count, 3)
))

cat("\nTotal IV (jointly optimized):", round(result_jedi$total_iv, 4), "\n")
cat("Converged:", result_jedi$converged, "\n")
cat("Iterations:", result_jedi$iterations, "\n")

# Example 2: Method comparison (JEDI vs alternatives)
set.seed(123)
n_obs_comp <- 2000

departments <- c(
  "Sales", "IT", "HR", "Finance", "Operations",
  "Marketing", "Legal", "R&D"
)
cat_feature_comp <- sample(departments, n_obs_comp, replace = TRUE)
bin_target_comp <- rbinom(n_obs_comp, 1, 0.12)

```

```

# JEDI (joint optimization)
result_jedi_comp <- ob_categorical_jedi(
  cat_feature_comp, bin_target_comp,
  min_bins = 3, max_bins = 4
)

# IVB (exact DP)
result_ivb_comp <- ob_categorical_ivb(
  cat_feature_comp, bin_target_comp,
  min_bins = 3, max_bins = 4
)

# GMB (greedy)
result_gmb_comp <- ob_categorical_gmb(
  cat_feature_comp, bin_target_comp,
  min_bins = 3, max_bins = 4
)

cat("\nMethod comparison (Total IV):\n")
cat(
  "  JEDI:", round(result_jedi_comp$total_iv, 4),
  "- converged:", result_jedi_comp$converged, "\n"
)
cat(
  "  IVB:", round(result_ivb_comp$total_iv, 4),
  "- converged:", result_ivb_comp$converged, "\n"
)
cat(
  "  GMB:", round(result_gmb_comp$total_iv, 4),
  "- converged:", result_gmb_comp$converged, "\n"
)

# Example 3: Bayesian smoothing with sparse data
set.seed(789)
n_obs_sparse <- 400

# Small sample with rare events
categories <- c("A", "B", "C", "D", "E", "F", "G")
cat_probs <- c(0.25, 0.20, 0.18, 0.15, 0.12, 0.07, 0.03)

cat_feature_sparse <- sample(categories, n_obs_sparse,
  replace = TRUE,
  prob = cat_probs
)
bin_target_sparse <- rbinom(n_obs_sparse, 1, 0.05) # 5% event rate

result_jedi_sparse <- ob_categorical_jedi(
  cat_feature_sparse,
  bin_target_sparse,
  min_bins = 2,
  max_bins = 4,
  bin_cutoff = 0.02
)

```

```

cat("\nBayesian smoothing (sparse data):\n")
cat(" Sample size:", n_obs_sparse, "\n")
cat(" Total events:", sum(bin_target_sparse), "\n")
cat(" Event rate:", round(mean(bin_target_sparse), 4), "\n")
cat(" Bins created:", length(result_jedi_sparse$bin), "\n\n")

# Show how smoothing prevents extreme WoE values
for (i in seq_along(result_jedi_sparse$bin)) {
  cat(sprintf(
    " Bin %d: events=%d/%d, WoE=% .3f (smoothed)\n",
    i,
    result_jedi_sparse$count_pos[i],
    result_jedi_sparse$count[i],
    result_jedi_sparse$woe[i]
  ))
}

# Example 4: Violation detection and repair
set.seed(456)
n_obs_viol <- 1200

# Create feature with non-monotonic risk pattern
risk_categories <- c(
  "VeryLow", "Low", "MediumHigh", "Medium", # Intentional non-monotonic
  "High", "VeryHigh"
)
actual_risks <- c(0.02, 0.05, 0.20, 0.12, 0.25, 0.40) # MediumHigh > Medium

cat_feature_viol <- sample(risk_categories, n_obs_viol, replace = TRUE)
bin_target_viol <- sapply(cat_feature_viol, function(x) {
  rbinom(1, 1, actual_risks[which(risk_categories == x)])
})

result_jedi_viol <- ob_categorical_jedi(
  cat_feature_viol,
  bin_target_viol,
  min_bins = 3,
  max_bins = 5,
  max_iterations = 50
)

cat("\nViolation detection and repair:\n")
cat(" Original categories:", length(unique(cat_feature_viol)), "\n")
cat(" Final bins:", length(result_jedi_viol$bin), "\n")
cat(" Iterations to convergence:", result_jedi_viol$iterations, "\n")
cat(" Monotonicity achieved:", result_jedi_viol$converged, "\n\n")

# Check final WoE monotonicity
woe_diffs <- diff(result_jedi_viol$woe)
cat(
  " WoE differences between bins:",
  paste(round(woe_diffs, 3), collapse = ", "), "\n"
)

```

```

)
cat(" All positive (monotonic):", all(woe_diffs >= -1e-6), "\n")

# Example 5: High cardinality performance
set.seed(321)
n_obs_hc <- 3000

# Simulate product categories (high cardinality)
products <- paste0("Product_", sprintf("%03d", 1:50))

cat_feature_hc <- sample(products, n_obs_hc, replace = TRUE)
bin_target_hc <- rbinom(n_obs_hc, 1, 0.08)

# Measure JEDI performance
time_jedi_hc <- system.time({
  result_jedi_hc <- ob_categorical_jedi(
    cat_feature_hc,
    bin_target_hc,
    min_bins = 4,
    max_bins = 7,
    max_n_prebins = 20,
    bin_cutoff = 0.02
  )
})

cat("\nHigh cardinality performance:\n")
cat(" Original categories:", length(unique(cat_feature_hc)), "\n")
cat(" Final bins:", length(result_jedi_hc$bin), "\n")
cat(" Execution time:", round(time_jedi_hc[3], 3), "seconds\n")
cat(" Total IV:", round(result_jedi_hc$total_iv, 4), "\n")
cat(" Converged:", result_jedi_hc$converged, "\n")

# Show merged categories
for (i in seq_along(result_jedi_hc$bin)) {
  n_merged <- length(strsplit(result_jedi_hc$bin[i], "%;%")[[1]])
  if (n_merged > 1) {
    cat(sprintf(" Bin %d: %d categories merged\n", i, n_merged))
  }
}

# Example 6: Convergence behavior
set.seed(555)
n_obs_conv <- 1000

education_levels <- c(
  "Elementary", "HighSchool", "Vocational",
  "Bachelor", "Master", "PhD"
)

cat_feature_conv <- sample(education_levels, n_obs_conv,
  replace = TRUE,
  prob = c(0.10, 0.30, 0.20, 0.25, 0.12, 0.03)
)

```

```
bin_target_conv <- rbinom(n_obs_conv, 1, 0.15)

# Test different convergence thresholds
thresholds <- c(1e-3, 1e-6, 1e-9)

for (thresh in thresholds) {
  result_conv <- ob_categorical_jedi(
    cat_feature_conv,
    bin_target_conv,
    min_bins = 2,
    max_bins = 4,
    convergence_threshold = thresh,
    max_iterations = 100
  )

  cat(sprintf("\nThreshold %.0e:\n", thresh))
  cat(" Final bins:", length(result_conv$bin), "\n")
  cat(" Total IV:", round(result_conv$total_iv, 4), "\n")
  cat(" Converged:", result_conv$converged, "\n")
  cat(" Iterations:", result_conv$iterations, "\n")
}

# Example 7: Missing value handling
set.seed(999)
cat_feature_na <- cat_feature
na_indices <- sample(n_obs, 75) # 5% missing
cat_feature_na[na_indices] <- NA

result_jedi_na <- ob_categorical_jedi(
  cat_feature_na,
  bin_target,
  min_bins = 3,
  max_bins = 5
)

# Locate NA bin
na_bin_idx <- grep("NA", result_jedi_na$bin)
if (length(na_bin_idx) > 0) {
  cat("\nMissing value treatment:\n")
  cat(" NA bin:", result_jedi_na$bin[na_bin_idx], "\n")
  cat(" NA count:", result_jedi_na$count[na_bin_idx], "\n")
  cat(
    " NA event rate:",
    round(result_jedi_na$count_pos[na_bin_idx] /
      result_jedi_na$count[na_bin_idx], 3), "\n"
  )
  cat(" NA WoE:", round(result_jedi_na$woe[na_bin_idx], 3), "\n")
  cat(" NA IV contribution:", round(result_jedi_na$iv[na_bin_idx], 4), "\n")
}
```

---

ob\_categorical\_jedi\_mwoe

*Optimal Binning for Categorical Variables with Multinomial Target  
using JEDI-MWoE*

---

## Description

Performs supervised discretization of categorical variables for multinomial classification problems using the Joint Entropy-Driven Information Maximization with Multinomial Weight of Evidence (JEDI-MWoE) algorithm. This advanced method extends traditional binning to handle multi-class targets through specialized information-theoretic measures and intelligent optimization strategies.

## Usage

```
ob_categorical_jedi_mwoe(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the special category "N/A".
target	An integer vector representing the multinomial outcome variable with consecutive integer classes starting from 0 (e.g., 0, 1, 2, ...). Missing values are not permitted. Must contain at least 2 distinct classes.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 1$ . The algorithm will not merge below this threshold. Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm iteratively merges until this constraint is satisfied. Defaults to 5.
bin_cutoff	Numeric. Minimum proportion of total observations required for a category to remain separate during initialization. Categories below this threshold are pre-merged. Must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. Maximum number of initial bins before the main optimization phase. Controls computational complexity for high-cardinality features. Must be $\geq \text{min\_bins}$ . Defaults to 20.

bin_separator	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
convergence_threshold	Numeric. Convergence tolerance based on Information Value change between iterations. Algorithm stops when $\max_c  \Delta IV_c  < \text{convergence\_threshold}$ across all classes. Must be > 0. Defaults to 1e-6.
max_iterations	Integer. Maximum number of optimization iterations. Prevents infinite loops in edge cases. Must be > 0. Defaults to 1000.

## Details

The JEDI-MWoE (Joint Entropy-Driven Information Maximization with Multinomial Weight of Evidence) algorithm extends traditional optimal binning to handle multinomial classification problems by computing class-specific information measures and optimizing joint information content across all target classes.

### Algorithm Workflow:

1. Input validation and preprocessing (multinomial target verification)
2. Initial bin creation (one category per bin)
3. Rare category merging (frequencies < bin\_cutoff)
4. Pre-bin limitation via statistical similarity merging
5. Main optimization loop with alternating strategies:
  - Jensen-Shannon divergence minimization for similar bin detection
  - Adjacent bin merging with minimal information loss
  - Class-wise monotonicity violation detection and repair
  - Convergence monitoring across all classes
6. Final constraint satisfaction (max\_bins enforcement)
7. Laplace-smoothed metric computation

### Multinomial Weight of Evidence (M-WoE):

For a bin  $B$  and class  $c$ , the Multinomial Weight of Evidence is:

$$M\text{-}WoE_{B,c} = \ln \left( \frac{P(c|B) + \alpha}{P(\neg c|B) + \alpha} \right)$$

where:

- $P(c|B) = \frac{n_{B,c}}{n_B}$  is the class probability in bin  $B$
- $P(\neg c|B) = \frac{\sum_{k \neq c} n_{B,k}}{\sum_{k \neq c} n_k}$  is the combined probability of all other classes in bin  $B$
- $\alpha = 0.5$  is the Laplace smoothing parameter

### Information Value Extension:

The Information Value for class  $c$  in bin  $B$  is:

$$IV_{B,c} = (P(c|B) - P(\neg c|B)) \times M\text{-}WoE_{B,c}$$

Total IV for class  $c$  across all bins:

$$IV_c = \sum_B |IV_{B,c}|$$

#### Statistical Similarity Measure:

JEDI-MWoE uses Jensen-Shannon divergence to identify similar bins for merging:

$$JS(P_B, P_{B'}) = \frac{1}{2} \sum_{c=0}^{C-1} \left[ P(c|B) \ln \frac{P(c|B)}{M(c)} + P(c|B') \ln \frac{P(c|B')}{M(c)} \right]$$

where  $M(c) = \frac{1}{2}[P(c|B) + P(c|B')]$  is the average distribution.

#### Class-wise Monotonicity Enforcement:

For each class  $c$ , the algorithm enforces WoE monotonicity by detecting violations (peaks and valleys) and repairing them through strategic bin merges:

- **Peak:**  $M\text{-WoE}_{i-1,c} < M\text{-WoE}_{i,c} > M\text{-WoE}_{i+1,c}$
- **Valley:**  $M\text{-WoE}_{i-1,c} > M\text{-WoE}_{i,c} < M\text{-WoE}_{i+1,c}$

Violation severity is measured as:

$$severity_{i,c} = \max\{|M\text{-WoE}_{i,c} - M\text{-WoE}_{i-1,c}|, |M\text{-WoE}_{i,c} - M\text{-WoE}_{i+1,c}|\}$$

#### Alternating Optimization Strategies:

The algorithm alternates between two merging strategies to balance global similarity and local information preservation:

1. **Divergence-based:** Merge bins with minimum JS divergence
2. **IV-preserving:** Merge adjacent bins with minimum information loss

#### Laplace Smoothing:

To ensure numerical stability and prevent undefined logarithms, all probability estimates are smoothed with a Laplace prior:

$$P_{smooth}(c|B) = \frac{n_{B,c} + \alpha}{n_B + \alpha \cdot C}$$

where  $C$  is the number of classes and  $\alpha = 0.5$ .

#### Computational Complexity:

- Time:  $O(k^2 \cdot C \cdot m)$  where  $k$  = bins,  $C$  = classes,  $m$  = iterations
- Space:  $O(k^2 \cdot C)$  for M-WoE cache
- Cache hit rate typically  $> 60\%$  for  $k > 10$

#### Key Innovations:

- **Multinomial extension:** Generalizes WoE/IV to multi-class problems

- **Joint optimization:** Simultaneously optimizes across all classes
- **Alternating strategies:** Balances global similarity and local preservation
- **Class-wise monotonicity:** Enforces meaningful ordering for each class
- **Statistical similarity:** Uses Jensen-Shannon divergence for merging

#### Comparison with Binary Methods:

Aspect	Binary	Multinomial	Extension
Target Classes	2	$C \geq 2$	One-vs-rest
WoE Definition	$\ln(p/n)$	$\ln(P(c)/P(\neg c))$	Class-specific
IV Aggregation	Sum	Per-class	Vector-valued
Similarity	Chi-square	Jensen-Shannon	Distribution-based
Monotonicity	Global	Per-class	Multi-constraint

#### Value

A list containing the multinomial binning results with the following components:

- id Integer vector of bin identifiers (1-indexed)
- bin Character vector of bin labels (merged category names)
- woe Numeric matrix of Multinomial Weight of Evidence values with dimensions (bins  $\times$  classes)
- iv Numeric matrix of Information Value contributions with dimensions (bins  $\times$  classes)
- count Integer vector of total observations per bin
- class\_counts Integer matrix of observations per class per bin with dimensions (bins  $\times$  classes)
- class\_rates Numeric matrix of class proportions per bin with dimensions (bins  $\times$  classes)
- converged Logical indicating algorithm convergence
- iterations Integer count of optimization iterations performed
- n\_classes Integer number of target classes
- total\_iv Numeric vector of total Information Value per class

#### References

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**See Also**

[ob\\_categorical\\_jedi](#) for binary target JEDI algorithm, [ob\\_categorical\\_ivb](#) for binary Information Value DP optimization, [ob\\_categorical\\_dp](#) for general dynamic programming binning

**Examples**

```

# Example 1: Basic multinomial JEDI-MWoE optimization
set.seed(42)
n_obs <- 1500

# Simulate customer segments with 3 risk categories
segments <- c("Premium", "Standard", "Basic", "Economy")
# Class probabilities: 0=LowRisk, 1=MediumRisk, 2=HighRisk
risk_probs <- list(
  Premium = c(0.80, 0.15, 0.05), # Mostly LowRisk
  Standard = c(0.40, 0.40, 0.20), # Balanced
  Basic = c(0.15, 0.35, 0.50), # Mostly HighRisk
  Economy = c(0.05, 0.20, 0.75) # Almost all HighRisk
)

cat_feature <- sample(segments, n_obs,
  replace = TRUE,
  prob = c(0.25, 0.35, 0.25, 0.15)
)

# Generate multinomial target (classes 0, 1, 2)
multinom_target <- sapply(cat_feature, function(segment) {
  probs <- risk_probs[[segment]]
  sample(0:2, 1, prob = probs)
})

# Apply JEDI-MWoE algorithm
result_mwoe <- ob_categorical_jedi_mwoe(
  cat_feature,
  multinom_target,
  min_bins = 2,
  max_bins = 3
)

# Display results
cat("Number of classes:", result_mwoe$n_classes, "\n")
cat("Number of bins:", length(result_mwoe$bin), "\n")
cat("Converged:", result_mwoe$converged, "\n")
cat("Iterations:", result_mwoe$iterations, "\n\n")

# Show bin details
for (i in seq_along(result_mwoe$bin)) {
  cat(sprintf("Bin %d (%s):\n", i, result_mwoe$bin[i]))
  cat("  Total count:", result_mwoe$count[i], "\n")
  cat("  Class counts:", result_mwoe$class_counts[i, ], "\n")
  cat("  Class rates:", round(result_mwoe$class_rates[i, ], 3), "\n")
}

```

```

# Show WoE and IV for each class
for (class in 0:(result_mwoe$n_classes - 1)) {
  cat(sprintf(
    " Class %d: WoE=%.3f, IV=%.4f\n",
    class,
    result_mwoe$woe[i, class + 1], # R is 1-indexed
    result_mwoe$iv[i, class + 1]
  ))
}
cat("\n")
}

# Show total IV per class
cat("Total IV per class:\n")
for (class in 0:(result_mwoe$n_classes - 1)) {
  cat(sprintf(" Class %d: %.4f\n", class, result_mwoe$total_iv[class + 1]))
}

# Example 2: High-cardinality multinomial problem
set.seed(123)
n_obs_hc <- 2000

# Simulate product categories with 4 classes
products <- paste0("Product_", LETTERS[1:15])
cat_feature_hc <- sample(products, n_obs_hc, replace = TRUE)

# Generate 4-class target
multinom_target_hc <- sample(0:3, n_obs_hc,
  replace = TRUE,
  prob = c(0.3, 0.25, 0.25, 0.2)
)

result_mwoe_hc <- ob_categorical_jedi_mwoe(
  cat_feature_hc,
  multinom_target_hc,
  min_bins = 3,
  max_bins = 6,
  max_n_prebins = 15,
  bin_cutoff = 0.03
)

cat("\nHigh-cardinality example:\n")
cat("Original categories:", length(unique(cat_feature_hc)), "\n")
cat("Final bins:", length(result_mwoe_hc$bin), "\n")
cat("Classes:", result_mwoe_hc$n_classes, "\n")
cat("Converged:", result_mwoe_hc$converged, "\n\n")

# Show merged categories
for (i in seq_along(result_mwoe_hc$bin)) {
  n_merged <- length(strsplit(result_mwoe_hc$bin[i], "%;%")[[1]])
  if (n_merged > 1) {
    cat(sprintf("Bin %d: %d categories merged\n", i, n_merged))
  }
}

```

```

}

# Example 3: Laplace smoothing demonstration
set.seed(789)
n_obs_smooth <- 500

# Small sample with sparse categories
categories <- c("A", "B", "C", "D", "E")
cat_feature_smooth <- sample(categories, n_obs_smooth,
  replace = TRUE,
  prob = c(0.3, 0.25, 0.2, 0.15, 0.1)
)

# Generate 3-class target with class imbalance
multinom_target_smooth <- sample(0:2, n_obs_smooth,
  replace = TRUE,
  prob = c(0.6, 0.3, 0.1)
) # Class 0 dominant

result_mwoe_smooth <- ob_categorical_jedi_mwoe(
  cat_feature_smooth,
  multinom_target_smooth,
  min_bins = 2,
  max_bins = 4,
  bin_cutoff = 0.02
)

cat("\nLaplace smoothing demonstration:\n")
cat("Sample size:", n_obs_smooth, "\n")
cat("Classes:", result_mwoe_smooth$n_classes, "\n")
cat("Event distribution:", table(multinom_target_smooth), "\n\n")

# Show how smoothing prevents extreme values
for (i in seq_along(result_mwoe_smooth$bin)) {
  cat(sprintf("Bin %d (%s):\n", i, result_mwoe_smooth$bin[i]))
  cat(" Counts per class:", result_mwoe_smooth$class_counts[i, ], "\n")
  cat(" WoE values:", round(result_mwoe_smooth$woe[i, ], 3), "\n")
  cat(" Note: Extreme WoE values prevented by Laplace smoothing\n\n")
}

# Example 4: Class-wise monotonicity
set.seed(456)
n_obs_mono <- 1200

# Feature with predictable class patterns
education <- c("PhD", "Master", "Bachelor", "College", "HighSchool")
# Each education level has a preferred class
preferred_classes <- c(2, 1, 0, 1, 2) # PhD→High(2), Bachelor→Low(0), etc.

cat_feature_mono <- sample(education, n_obs_mono, replace = TRUE)

# Generate target with preferred class bias
multinom_target_mono <- sapply(cat_feature_mono, function(edu) {

```

```

pref_class <- preferred_classes[which(education == edu)]
# Create probability vector with preference
probs <- rep(0.1, 3) # Base probability
probs[pref_class + 1] <- 0.8 # Preferred class gets high probability
sample(0:2, 1, prob = probs / sum(probs))
})

result_mwoe_mono <- ob_categorical_jedi_mwoe(
  cat_feature_mono,
  multinom_target_mono,
  min_bins = 3,
  max_bins = 5
)

cat("Class-wise monotonicity example:\n")
cat("Education levels:", length(education), "\n")
cat("Final bins:", length(result_mwoe_mono$bin), "\n")
cat("Iterations:", result_mwoe_mono$iterations, "\n\n")

# Check monotonicity for each class
for (class in 0:(result_mwoe_mono$n_classes - 1)) {
  woe_series <- result_mwoe_mono$woe[, class + 1]
  diffs <- diff(woe_series)
  is_mono <- all(diffs >= -1e-6) || all(diffs <= 1e-6)
  cat(sprintf("Class %d WoE monotonic: %s\n", class, is_mono))
  cat(sprintf(" WoE series: %s\n", paste(round(woe_series, 3), collapse = ", ")))
}

# Example 5: Missing value handling
set.seed(321)
cat_feature_na <- cat_feature
na_indices <- sample(n_obs, 75) # 5% missing
cat_feature_na[na_indices] <- NA

result_mwoe_na <- ob_categorical_jedi_mwoe(
  cat_feature_na,
  multinom_target,
  min_bins = 2,
  max_bins = 3
)

# Locate missing value bin
missing_bin_idx <- grep("N/A", result_mwoe_na$bin)
if (length(missing_bin_idx) > 0) {
  cat("\nMissing value handling:\n")
  cat("Missing value bin:", result_mwoe_na$bin[missing_bin_idx], "\n")
  cat("Missing value count:", result_mwoe_na$count[missing_bin_idx], "\n")
  cat(
    "Class distribution in missing bin:",
    result_mwoe_na$class_counts[missing_bin_idx, ], "\n"
  )
}

# Show class rates for missing bin

```

```

for (class in 0:(result_mwoe_na$n_classes - 1)) {
  cat(sprintf(
    "  Class %d rate: %.3f\n", class,
    result_mwoe_na$class_rates[missing_bin_idx, class + 1]
  ))
}
}

# Example 6: Convergence behavior
set.seed(555)
n_obs_conv <- 1000

departments <- c("Sales", "IT", "HR", "Finance", "Operations")
cat_feature_conv <- sample(departments, n_obs_conv, replace = TRUE)
multinom_target_conv <- sample(0:2, n_obs_conv, replace = TRUE)

# Test different convergence thresholds
thresholds <- c(1e-3, 1e-6, 1e-9)

for (thresh in thresholds) {
  result_conv <- ob_categorical_jedi_mwoe(
    cat_feature_conv,
    multinom_target_conv,
    min_bins = 2,
    max_bins = 4,
    convergence_threshold = thresh,
    max_iterations = 100
  )

  cat(sprintf("\nThreshold %.0e:\n", thresh))
  cat("  Final bins:", length(result_conv$bin), "\n")
  cat("  Converged:", result_conv$converged, "\n")
  cat("  Iterations:", result_conv$iterations, "\n")

  # Show total IV for each class
  cat("  Total IV per class:")
  for (class in 0:(result_conv$n_classes - 1)) {
    cat(sprintf(" %.4f", result_conv$total_iv[class + 1]))
  }
  cat("\n")
}

```

## Description

Performs supervised discretization of categorical variables using the Monotonic Binning Algorithm (MBA), which enforces strict Weight of Evidence monotonicity while optimizing Information Value through intelligent bin merging strategies. This implementation includes Bayesian smoothing for numerical stability and adaptive thresholding for robust monotonicity enforcement.

## Usage

```
ob_categorical_mba(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable to be binned. Missing values are automatically converted to the category "NA".
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Missing values are not permitted.
min_bins	Integer. Minimum number of bins to produce. Must be $\geq 2$ . The algorithm will not merge below this threshold. Defaults to 3.
max_bins	Integer. Maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . The algorithm reduces bins until this constraint is met. Defaults to 5.
bin_cutoff	Numeric. Minimum proportion of total observations required for a category to remain separate. Categories below this threshold are pre-merged with similar categories. Must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. Maximum number of initial bins before the main optimization phase. Controls computational complexity. Must be $\geq \text{max\_bins}$ . Defaults to 20.
bin_separator	Character string used to concatenate category names when multiple categories are merged into a single bin. Defaults to "%;%".
convergence_threshold	Numeric. Convergence tolerance based on Information Value change between iterations. Algorithm stops when $ \Delta IV  < \text{convergence\_threshold}$ . Must be $> 0$ . Defaults to 1e-6.
max_iterations	Integer. Maximum number of optimization iterations. Prevents infinite loops. Must be $> 0$ . Defaults to 1000.

## Details

The Monotonic Binning Algorithm (MBA) implements a sophisticated approach to categorical binning that guarantees strict Weight of Evidence monotonicity through intelligent violation detection and repair mechanisms.

### Algorithm Workflow:

1. Input validation and preprocessing
2. Initial bin creation (one category per bin)
3. Pre-binning limitation to `max_n_prebins`
4. Rare category merging (frequencies < `bin_cutoff`)
5. Bayesian-smoothed WoE calculation
6. Strict monotonicity enforcement with adaptive thresholds
7. IV-optimized bin merging to meet `max_bins` constraint
8. Final consistency verification

### Monotonicity Enforcement:

MBA enforces strict monotonicity through an iterative repair process:

1. Sort bins by current WoE values
2. Calculate adaptive threshold:  $\tau = \min(\epsilon, 0.01\bar{\Delta})$
3. Identify violations:  $\text{sign}(WoE_i - WoE_{i-1}) \neq \text{sign}(WoE_{i+1} - WoE_i)$
4. Rank violations by severity:  $s_i = |WoE_i - WoE_{i-1}| + |WoE_{i+1} - WoE_i|$
5. Repair most severe violations by merging adjacent bins
6. Repeat until no violations remain or `min_bins` reached

### Bayesian Smoothing:

To ensure numerical stability and prevent overfitting, MBA applies Bayesian smoothing to WoE and IV calculations:

$$p'_i = \frac{n_{i,\text{pos}} + \alpha_p}{N_{\text{pos}} + \alpha_{\text{total}}}$$

$$n'_i = \frac{n_{i,\text{neg}} + \alpha_n}{N_{\text{neg}} + \alpha_{\text{total}}}$$

where priors are proportional to overall prevalence:

$$\alpha_p = \alpha_{\text{total}} \times \frac{N_{\text{pos}}}{N_{\text{pos}} + N_{\text{neg}}}$$

$$\alpha_n = \alpha_{\text{total}} - \alpha_p$$

with  $\alpha_{\text{total}} = 1.0$  as the prior strength parameter.

### Intelligent Bin Merging:

When reducing bins to meet the `max_bins` constraint, MBA employs an IV-loss minimization strategy:

$$\Delta IV_{i,j} = IV_i + IV_j - IV_{merged}(i,j)$$

The pair with minimum  $\Delta IV$  is merged to preserve maximum predictive information.

#### Computational Complexity:

- Time:  $O(k^2 \cdot m)$  where  $k$  = bins,  $m$  = iterations
- Space:  $O(k^2)$  for IV loss cache
- Cache hit rate typically  $> 75\%$  for  $k > 10$

#### Key Features:

- **Guaranteed monotonicity:** Strict enforcement with adaptive thresholds
- **Bayesian regularization:** Robust to sparse bins and class imbalance
- **Intelligent merging:** Preserves maximum information during reduction
- **Adaptive thresholds:** Context-aware violation detection
- **Consistency verification:** Final integrity checks

#### Value

A list containing the binning results with the following components:

- id Integer vector of bin identifiers (1-indexed)
- bin Character vector of bin labels (merged category names)
- woe Numeric vector of Weight of Evidence values per bin
- iv Numeric vector of Information Value contribution per bin
- count Integer vector of total observations per bin
- count\_pos Integer vector of positive cases (target=1) per bin
- count\_neg Integer vector of negative cases (target=0) per bin
- total\_iv Numeric total Information Value of the binning solution
- converged Logical indicating algorithm convergence
- iterations Integer count of optimization iterations performed

#### References

Mironchyk, P., & Tchistiakov, V. (2017). Monotone optimal binning algorithm for credit risk modeling. *SSRN Electronic Journal*. doi:10.2139/ssrn.2978774

Siddiqi, N. (2017). *Intelligent Credit Scoring: Building and Implementing Better Credit Risk Scorecards* (2nd ed.). Wiley.

Good, I. J. (1965). *The Estimation of Probabilities: An Essay on Modern Bayesian Methods*. MIT Press.

Zeng, G. (2014). A necessary condition for a good binning algorithm in credit scoring. *Applied Mathematical Sciences*, 8(65), 3229-3242.

**See Also**

[ob\\_categorical\\_jedi](#) for joint entropy-driven optimization, [ob\\_categorical\\_dp](#) for dynamic programming approach, [ob\\_categorical\\_cm](#) for ChiMerge-based binning

**Examples**

```

# Example 1: Basic monotonic binning with guaranteed WoE ordering
set.seed(42)
n_obs <- 1500

# Simulate risk ratings with natural monotonic relationship
ratings <- c("AAA", "AA", "A", "BBB", "BB", "B", "CCC")
default_probs <- c(0.01, 0.02, 0.05, 0.10, 0.20, 0.35, 0.50)

cat_feature <- sample(ratings, n_obs,
  replace = TRUE,
  prob = c(0.05, 0.10, 0.20, 0.25, 0.20, 0.15, 0.05)
)
bin_target <- sapply(cat_feature, function(x) {
  rbinom(1, 1, default_probs[which(ratings == x)])
})

# Apply MBA algorithm
result_mba <- ob_categorical_mba(
  cat_feature,
  bin_target,
  min_bins = 3,
  max_bins = 5
)

# Display results with guaranteed monotonic WoE
print(data.frame(
  Bin = result_mba$bin,
  WoE = round(result_mba$woe, 3),
  IV = round(result_mba$iv, 4),
  Count = result_mba$count,
  EventRate = round(result_mba$count_pos / result_mba$count, 3)
))

cat("\nMonotonicity check (WoE differences):\n")
woe_diffs <- diff(result_mba$woe)
cat(" Differences:", paste(round(woe_diffs, 4), collapse = ", "), "\n")
cat(" All positive (increasing):", all(woe_diffs >= -1e-10), "\n")
cat(" Total IV:", round(result_mba$total_iv, 4), "\n")
cat(" Converged:", result_mba$converged, "\n")

# Example 2: Comparison with non-monotonic methods
set.seed(123)
n_obs_comp <- 2000

sectors <- c("Tech", "Health", "Finance", "Manufacturing", "Retail")
cat_feature_comp <- sample(sectors, n_obs_comp, replace = TRUE)

```

```

bin_target_comp <- rbinom(n_obs_comp, 1, 0.15)

# MBA (strictly monotonic)
result_mba_comp <- ob_categorical_mba(
  cat_feature_comp, bin_target_comp,
  min_bins = 3, max_bins = 4
)

# Standard binning (may not be monotonic)
result_std_comp <- ob_categorical_cm(
  cat_feature_comp, bin_target_comp,
  min_bins = 3, max_bins = 4
)

cat("\nMonotonicity comparison:\n")
cat(
  "  MBA WoE differences:",
  paste(round(diff(result_mba_comp$woe), 4), collapse = ", "), "\n"
)
cat("  MBA monotonic:", all(diff(result_mba_comp$woe) >= -1e-10), "\n")
cat(
  "  Std WoE differences:",
  paste(round(diff(result_std_comp$woe), 4), collapse = ", "), "\n"
)
cat("  Std monotonic:", all(diff(result_std_comp$woe) >= -1e-10), "\n")

# Example 3: Bayesian smoothing with sparse data
set.seed(789)
n_obs_sparse <- 400

# Small sample with rare categories
categories <- c("A", "B", "C", "D", "E", "F")
cat_probs <- c(0.30, 0.25, 0.20, 0.15, 0.07, 0.03)

cat_feature_sparse <- sample(categories, n_obs_sparse,
  replace = TRUE,
  prob = cat_probs
)
bin_target_sparse <- rbinom(n_obs_sparse, 1, 0.08) # 8% event rate

result_mba_sparse <- ob_categorical_mba(
  cat_feature_sparse,
  bin_target_sparse,
  min_bins = 2,
  max_bins = 4,
  bin_cutoff = 0.02
)

cat("\nBayesian smoothing (sparse data):\n")
cat("  Sample size:", n_obs_sparse, "\n")
cat("  Events:", sum(bin_target_sparse), "\n")
cat("  Final bins:", length(result_mba_sparse$bin), "\n\n")

```

```

# Show how smoothing prevents extreme WoE values
for (i in seq_along(result_mba_sparse$bin)) {
  cat(sprintf(
    " Bin %d: events=%d/%d, WoE=% .3f (smoothed)\n",
    i,
    result_mba_sparse$count_pos[i],
    result_mba_sparse$count[i],
    result_mba_sparse$woe[i]
  ))
}

# Example 4: High cardinality with pre-binning
set.seed(456)
n_obs_hc <- 3000

# Simulate ZIP codes (high cardinality)
zips <- paste0("ZIP_", sprintf("%04d", 1:50))

cat_feature_hc <- sample(zips, n_obs_hc, replace = TRUE)
bin_target_hc <- rbinom(n_obs_hc, 1, 0.12)

result_mba_hc <- ob_categorical_mba(
  cat_feature_hc,
  bin_target_hc,
  min_bins = 4,
  max_bins = 6,
  max_n_prebins = 20,
  bin_cutoff = 0.01
)

cat("\nHigh cardinality performance:\n")
cat(" Original categories:", length(unique(cat_feature_hc)), "\n")
cat(" Final bins:", length(result_mba_hc$bin), "\n")
cat(
  " Largest merged bin contains:",
  max(sapply(strsplit(result_mba_hc$bin, "%;%"),
             length)), "categories\n"
)

# Verify monotonicity in high-cardinality case
woe_monotonic <- all(diff(result_mba_hc$woe) >= -1e-10)
cat(" WoE monotonic:", woe_monotonic, "\n")

# Example 5: Convergence behavior
set.seed(321)
n_obs_conv <- 1000

business_sizes <- c("Micro", "Small", "Medium", "Large", "Enterprise")
cat_feature_conv <- sample(business_sizes, n_obs_conv, replace = TRUE)
bin_target_conv <- rbinom(n_obs_conv, 1, 0.18)

# Test different convergence thresholds
thresholds <- c(1e-3, 1e-6, 1e-9)

```

```

for (thresh in thresholds) {
  result_conv <- ob_categorical_mba(
    cat_feature_conv,
    bin_target_conv,
    min_bins = 2,
    max_bins = 4,
    convergence_threshold = thresh,
    max_iterations = 50
  )

  cat(sprintf("\nThreshold %.0e:\n", thresh))
  cat("  Final bins:", length(result_conv$bin), "\n")
  cat("  Total IV:", round(result_conv$total_iv, 4), "\n")
  cat("  Converged:", result_conv$converged, "\n")
  cat("  Iterations:", result_conv$iterations, "\n")

  # Check monotonicity preservation
  monotonic <- all(diff(result_conv$woe) >= -1e-10)
  cat("  Monotonic:", monotonic, "\n")
}

# Example 6: Missing value handling
set.seed(555)
cat_feature_na <- cat_feature
na_indices <- sample(n_obs, 75) # 5% missing
cat_feature_na[na_indices] <- NA

result_mba_na <- ob_categorical_mba(
  cat_feature_na,
  bin_target,
  min_bins = 3,
  max_bins = 5
)

# Locate NA bin
na_bin_idx <- grep("NA", result_mba_na$bin)
if (length(na_bin_idx) > 0) {
  cat("\nMissing value treatment:\n")
  cat("  NA bin:", result_mba_na$bin[na_bin_idx], "\n")
  cat("  NA count:", result_mba_na$count[na_bin_idx], "\n")
  cat(
    "  NA event rate:",
    round(result_mba_na$count_pos[na_bin_idx] /
      result_mba_na$count[na_bin_idx], 3), "\n"
  )
  cat("  NA WoE:", round(result_mba_na$woe[na_bin_idx], 3), "\n")
  cat(
    "  Monotonicity preserved:",
    all(diff(result_mba_na$woe) >= -1e-10), "\n"
  )
}

```

---

 ob\_categorical\_milp    *Optimal Binning for Categorical Variables using Heuristic Algorithm*


---

## Description

This function performs optimal binning for categorical variables using a heuristic merging approach to maximize Information Value (IV) while maintaining monotonic Weight of Evidence (WoE). Despite its name containing "MILP", it does NOT use Mixed Integer Linear Programming but rather a greedy optimization algorithm.

## Usage

```
ob_categorical_milp(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 20L,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000L
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature. Cannot contain missing values.
min_bins	Integer. Minimum number of bins to create. Must be at least 2. Default is 3.
max_bins	Integer. Maximum number of bins to create. Must be greater than or equal to min_bins. Default is 5.
bin_cutoff	Numeric. Minimum relative frequency threshold for individual categories. Categories with frequency below this proportion will be merged with others. Value must be between 0 and 1. Default is 0.05 (5%).
max_n_prebins	Integer. Maximum number of initial bins before optimization. Used to control computational complexity when dealing with high-cardinality categorical variables. Default is 20.
bin_separator	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on changes in total Information Value. Must be positive. Default is 1e-6.

`max_iterations` Integer. Maximum number of iterations for the optimization process. Must be positive. Default is 1000.

## Details

The algorithm follows these steps:

1. Pre-binning: Each unique category becomes an initial bin
2. Rare category handling: Categories below `bin_cutoff` frequency are merged with similar ones
3. Bin reduction: Greedily merge bins to satisfy `min_bins` and `max_bins` constraints
4. Monotonicity enforcement: Ensures WoE is either consistently increasing or decreasing across bins
5. Optimization: Iteratively improves Information Value

Key features include:

- Bayesian smoothing to stabilize WoE estimates for sparse categories
- Automatic handling of missing values (converted to "NA" category)
- Monotonicity constraint enforcement
- Configurable minimum and maximum bin counts
- Rare category pooling based on relative frequency thresholds

Mathematical definitions:

$$WoE_i = \ln \left( \frac{p_i^{(1)}}{p_i^{(0)}} \right)$$

where  $p_i^{(1)}$  and  $p_i^{(0)}$  are the proportions of positive and negative cases in bin  $i$ , respectively, adjusted using Bayesian smoothing.

$$IV = \sum_{i=1}^n (p_i^{(1)} - p_i^{(0)}) \times WoE_i$$

## Value

A list containing the results of the optimal binning procedure:

- `id` Integer vector of bin identifiers (1 to `n_bins`)
- `bin` Character vector of bin labels, which are combinations of original categories separated by `bin_separator`
- `woe` Numeric vector of Weight of Evidence values for each bin
- `iv` Numeric vector of Information Values for each bin
- `count` Integer vector of total observations in each bin
- `count_pos` Integer vector of positive outcomes in each bin
- `count_neg` Integer vector of negative outcomes in each bin
- `total_iv` Numeric scalar. Total Information Value across all bins
- `converged` Logical. Whether the algorithm converged within the specified tolerance
- `iterations` Integer. Number of iterations performed

**Note**

- Target variable must contain both 0 and 1 values.
- Empty strings in the feature vector are not allowed and will cause an error.
- For datasets with very few observations in either class (<5), warnings will be issued as results may be unstable.
- The algorithm uses a greedy heuristic approach, not true MILP optimization. For exact solutions, external solvers like Gurobi or CPLEX would be required.

**Examples**

```

# Generate sample data
set.seed(123)
n <- 1000
feature <- sample(letters[1:8], n, replace = TRUE)
target <- rbinom(n, 1, prob = ifelse(feature %in% c("a", "b"), 0.7, 0.3))

# Perform optimal binning
result <- ob_categorical_milp(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# With custom parameters
result2 <- ob_categorical_milp(
  feature = feature,
  target = target,
  min_bins = 2,
  max_bins = 4,
  bin_cutoff = 0.03
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 50)] <- NA
result3 <- ob_categorical_milp(feature_with_na, target)

```

---

ob_categorical_mob	<i>Optimal Binning for Categorical Variables using Monotonic Optimal Binning (MOB)</i>
--------------------	--

---

**Description**

This function performs optimal binning for categorical variables using the Monotonic Optimal Binning (MOB) algorithm. It creates bins that maintain monotonic Weight of Evidence (WoE) trends while maximizing Information Value.

## Usage

```
ob_categorical_mob(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 20L,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000L
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature. Cannot contain missing values.
min_bins	Integer. Minimum number of bins to create. Must be at least 1. Default is 3.
max_bins	Integer. Maximum number of bins to create. Must be greater than or equal to min_bins. Default is 5.
bin_cutoff	Numeric. Minimum relative frequency threshold for individual categories. Categories with frequency below this proportion will be merged with others. Value must be between 0 and 1. Default is 0.05 (5%).
max_n_prebins	Integer. Maximum number of initial bins before optimization. Used to control computational complexity when dealing with high-cardinality categorical variables. Default is 20.
bin_separator	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on changes in total Information Value. Must be positive. Default is 1e-6.
max_iterations	Integer. Maximum number of iterations for the optimization process. Must be positive. Default is 1000.

## Details

The MOB algorithm follows these steps:

1. Initial sorting: Categories are ordered by their individual WoE values
2. Rare category handling: Categories below bin\_cutoff frequency are merged with similar ones
3. Pre-binning limitation: Reduces initial bins to max\_n\_prebins using similarity-based merging

4. Monotonicity enforcement: Ensures WoE is either consistently increasing or decreasing across bins
5. Bin count optimization: Adjusts to meet `min_bins/max_bins` constraints

Key features include:

- Automatic sorting of categories by WoE for initial structure
- Bayesian smoothing to stabilize WoE estimates for sparse categories
- Guaranteed monotonic WoE trend across final bins
- Configurable minimum and maximum bin counts
- Similarity-based merging for optimal bin combinations

Mathematical definitions:

$$WoE_i = \ln \left( \frac{p_i^{(1)}}{p_i^{(0)}} \right)$$

where  $p_i^{(1)}$  and  $p_i^{(0)}$  are the proportions of positive and negative cases in bin  $i$ , respectively, adjusted using Bayesian smoothing.

$$IV = \sum_{i=1}^n (p_i^{(1)} - p_i^{(0)}) \times WoE_i$$

## Value

A list containing the results of the optimal binning procedure:

- `id` Numeric vector of bin identifiers (1 to `n_bins`)
- `bin` Character vector of bin labels, which are combinations of original categories separated by `bin_separator`
- `woe` Numeric vector of Weight of Evidence values for each bin
- `iv` Numeric vector of Information Values for each bin
- `count` Integer vector of total observations in each bin
- `count_pos` Integer vector of positive outcomes in each bin
- `count_neg` Integer vector of negative outcomes in each bin
- `total_iv` Numeric scalar. Total Information Value across all bins
- `converged` Logical. Whether the algorithm converged within the specified tolerance
- `iterations` Integer. Number of iterations performed

## Note

- Target variable must contain both 0 and 1 values.
- Empty strings in the feature vector are not allowed and will cause an error.
- For datasets with very few observations in either class (<5), warnings will be issued as results may be unstable.
- The algorithm guarantees monotonic WoE across bins.
- When the number of unique categories is less than `max_bins`, each category will form its own bin.

## Examples

```

# Generate sample data
set.seed(123)
n <- 1000
feature <- sample(letters[1:8], n, replace = TRUE)
target <- rbinom(n, 1, prob = ifelse(feature %in% c("a", "b"), 0.7, 0.3))

# Perform optimal binning
result <- ob_categorical_mob(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# With custom parameters
result2 <- ob_categorical_mob(
  feature = feature,
  target = target,
  min_bins = 2,
  max_bins = 4,
  bin_cutoff = 0.03
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 50)] <- NA
result3 <- ob_categorical_mob(feature_with_na, target)

```

---

ob\_categorical\_sab      *Optimal Binning for Categorical Variables using Simulated Annealing*

---

## Description

This function performs optimal binning for categorical variables using a Simulated Annealing (SA) optimization algorithm. It maximizes Information Value (IV) while maintaining monotonic Weight of Evidence (WoE) trends.

## Usage

```

ob_categorical_sab(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_preibins = 20L,
  bin_separator = "%;%",
  initial_temperature = 1,
  cooling_rate = 0.995,
  max_iterations = 1000L,

```

```

convergence_threshold = 1e-06,
adaptive_cooling = TRUE
)

```

## Arguments

<b>feature</b>	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
<b>target</b>	An integer vector containing binary outcome values (0 or 1). Must be the same length as <b>feature</b> . Cannot contain missing values.
<b>min_bins</b>	Integer. Minimum number of bins to create. Must be at least 2. Default is 3.
<b>max_bins</b>	Integer. Maximum number of bins to create. Must be greater than or equal to <b>min_bins</b> . Default is 5.
<b>bin_cutoff</b>	Numeric. Minimum relative frequency threshold for individual bins. Bins with frequency below this proportion will be penalized. Value must be between 0 and 1. Default is 0.05 (5%).
<b>max_n_prebins</b>	Integer. Maximum number of initial categories before optimization (not directly used in current implementation). Must be greater than or equal to <b>max_bins</b> . Default is 20.
<b>bin_separator</b>	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
<b>initial_temperature</b>	Numeric. Starting temperature for the simulated annealing algorithm. Higher values allow more exploration. Must be positive. Default is 1.0.
<b>cooling_rate</b>	Numeric. Rate at which temperature decreases during optimization. Value must be between 0 and 1. Lower values lead to faster cooling. Default is 0.995.
<b>max_iterations</b>	Integer. Maximum number of iterations for the optimization process. Must be positive. Default is 1000.
<b>convergence_threshold</b>	Numeric. Threshold for determining algorithm convergence based on changes in Information Value. Must be positive. Default is 1e-6.
<b>adaptive_cooling</b>	Logical. Whether to use adaptive cooling that modifies the cooling rate based on search progress. Default is TRUE.

## Details

The SAB (Simulated Annealing Binning) algorithm follows these steps:

1. Initialization: Categories are initially assigned to bins using a k-means-like strategy based on event rates
2. Optimization: Simulated annealing explores different bin assignments to maximize IV
3. Neighborhood generation: Multiple strategies are employed to generate neighboring solutions (swaps, reassignments, event-rate based moves)

4. Acceptance criteria: New solutions are accepted based on the Metropolis criterion with adaptive temperature control
5. Monotonicity enforcement: Final solutions are adjusted to ensure monotonic WoE trends

Key features include:

- Global optimization approach using simulated annealing
- Adaptive cooling schedule to balance exploration and exploitation
- Multiple neighborhood generation strategies for better search
- Bayesian smoothing to stabilize WoE estimates for sparse categories
- Guaranteed monotonic WoE trend across final bins
- Configurable optimization parameters for fine-tuning

Mathematical definitions:

$$WoE_i = \ln \left( \frac{p_i^{(1)}}{p_i^{(0)}} \right)$$

where  $p_i^{(1)}$  and  $p_i^{(0)}$  are the proportions of positive and negative cases in bin  $i$ , respectively, adjusted using Bayesian smoothing.

$$IV = \sum_{i=1}^n (p_i^{(1)} - p_i^{(0)}) \times WoE_i$$

The acceptance probability in simulated annealing is:

$$P(\text{accept}) = \exp \left( \frac{IV_{\text{new}} - IV_{\text{current}}}{T} \right)$$

where  $T$  is the current temperature.

## Value

A list containing the results of the optimal binning procedure:

- id Numeric vector of bin identifiers (1 to n\_bins)
- bin Character vector of bin labels, which are combinations of original categories separated by bin\_separator
- woe Numeric vector of Weight of Evidence values for each bin
- iv Numeric vector of Information Values for each bin
- count Integer vector of total observations in each bin
- count\_pos Integer vector of positive outcomes in each bin
- count\_neg Integer vector of negative outcomes in each bin
- total\_iv Numeric scalar. Total Information Value across all bins
- converged Logical. Whether the algorithm converged within the specified tolerance
- iterations Integer. Number of iterations performed

### Note

- Target variable must contain both 0 and 1 values.
- Empty strings in the feature vector are not allowed and will cause an error.
- For datasets with very few observations in either class (<5), warnings will be issued as results may be unstable.
- The algorithm uses global optimization which may require more computational time compared to heuristic approaches.
- When the number of unique categories is less than `max_bins`, each category will form its own bin.

### Examples

```

# Generate sample data
set.seed(123)
n <- 1000
feature <- sample(letters[1:8], n, replace = TRUE)
target <- rbinom(n, 1, prob = ifelse(feature %in% c("a", "b"), 0.7, 0.3))

# Perform optimal binning
result <- ob_categorical_sab(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# With custom parameters
result2 <- ob_categorical_sab(
  feature = feature,
  target = target,
  min_bins = 2,
  max_bins = 4,
  initial_temperature = 2.0,
  cooling_rate = 0.99
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 50)] <- NA
result3 <- ob_categorical_sab(feature_with_na, target)

```

### Description

This function performs optimal binning for categorical variables using the Similarity-Based Logistic Partitioning (SBLP) algorithm. This approach combines logistic properties (sorting categories by event rate) with dynamic programming to find the optimal partition that maximizes Information Value (IV).

## Usage

```
ob_categorical_sblp(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 20L,
  convergence_threshold = 1e-06,
  max_iterations = 1000L,
  bin_separator = "%;%",
  alpha = 0.5
)
```

## Arguments

feature	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature. Cannot contain missing values.
min_bins	Integer. Minimum number of bins to create. Must be at least 2. Default is 3.
max_bins	Integer. Maximum number of bins to create. Must be greater than or equal to min_bins. Default is 5.
bin_cutoff	Numeric. Minimum relative frequency threshold for individual categories. Categories with frequency below this proportion will be merged with similar categories before the main optimization. Value must be between 0 and 1. Default is 0.05 (5%).
max_n_prebins	Integer. Maximum number of initial bins/groups allowed before the dynamic programming optimization. If the number of unique categories exceeds this, similar adjacent categories are pre-merged. Default is 20.
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on changes in total Information Value. Default is 1e-6.
max_iterations	Integer. Maximum number of iterations for the optimization process. Default is 1000.
bin_separator	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
alpha	Numeric. Laplace smoothing parameter added to counts to avoid division by zero and stabilize WoE calculations for sparse data. Must be non-negative. Default is 0.5.

## Details

The SBLP algorithm follows these steps:

1. **Preprocessing:** Handling of missing values and calculation of initial statistics.
2. **Rare Category Consolidation:** Categories with frequency below bin\_cutoff are merged with statistically similar categories based on their target rates.
3. **Sorting:** Unique categories (or merged groups) are sorted by their empirical event rate (probability of target=1).
4. **Dynamic Programming:** An optimal partitioning algorithm (similar to Jenks Natural Breaks but optimizing IV) is applied to the sorted sequence to determine the cutpoints that maximize the total IV.
5. **Refinement:** Post-processing ensures constraints like monotonicity and minimum bin size are met.

A key feature of this implementation is the use of **Laplace Smoothing** (controlled by the alpha parameter) to prevent infinite WoE values and stabilize estimates for categories with small counts.

Mathematical definitions with smoothing:

The smoothed event rate  $p_i$  for a bin is calculated as:

$$p_i = \frac{n_{pos} + \alpha}{n_{total} + 2\alpha}$$

The Weight of Evidence (WoE) is computed using smoothed proportions:

$$WoE_i = \ln \left( \frac{p_i^{(1)}}{p_i^{(0)}} \right)$$

where  $p_i^{(1)}$  and  $p_i^{(0)}$  are the smoothed distributions of positive and negative classes across bins.

## Value

A list containing the results of the optimal binning procedure:

id Numeric vector of bin identifiers (1 to n\_bins)  
 bin Character vector of bin labels, which are combinations of original categories separated by bin\_separator  
 woe Numeric vector of Weight of Evidence values for each bin  
 iv Numeric vector of Information Values for each bin  
 count Integer vector of total observations in each bin  
 count\_pos Integer vector of positive outcomes in each bin  
 count\_neg Integer vector of negative outcomes in each bin  
 rate Numeric vector of the observed event rate in each bin  
 total\_iv Numeric scalar. Total Information Value across all bins  
 converged Logical. Whether the algorithm converged  
 iterations Integer. Number of iterations performed

## Note

- Target variable must contain both 0 and 1 values.
- Unlike heuristic methods, this algorithm uses Dynamic Programming which guarantees an optimal partition given the sorted order of categories.
- Monotonicity is generally enforced by the sorting step, but strictly checked and corrected in the final output.

## Examples

```
# Generate sample data
set.seed(123)
n <- 1000
feature <- sample(letters[1:8], n, replace = TRUE)
# Create a relationship where 'a' and 'b' have high probability
target <- rbinom(n, 1, prob = ifelse(feature %in% c("a", "b"), 0.8, 0.2))

# Perform optimal binning
result <- ob_categorical_sb1p(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# Using a higher smoothing parameter (alpha)
result_smooth <- ob_categorical_sb1p(
  feature = feature,
  target = target,
  alpha = 1.0
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 50)] <- NA
result_na <- ob_categorical_sb1p(feature_with_na, target)
```

---

ob\_categorical\_sketch *Optimal Binning for Categorical Variables using Sketch-based Algorithm*

---

## Description

This function performs optimal binning for categorical variables using a Sketch-based algorithm designed for large-scale data processing. It employs probabilistic data structures (Count-Min Sketch) to efficiently estimate category frequencies and event rates, enabling near real-time binning on massive datasets.

## Usage

```
ob_categorical_sketch(
  feature,
```

```

  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 20L,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000L,
  sketch_width = 2000L,
  sketch_depth = 5L
)

```

## Arguments

feature	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "N/A" and treated as a separate category.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature. Cannot contain missing values.
min_bins	Integer. Minimum number of bins to create. Must be at least 2. Default is 3.
max_bins	Integer. Maximum number of bins to create. Must be greater than or equal to min_bins. Default is 5.
bin_cutoff	Numeric. Minimum relative frequency threshold for categories to be considered "heavy hitters". Categories below this proportion will be grouped together. Value must be between 0 and 1. Default is 0.05 (5%).
max_n_prebins	Integer. Maximum number of initial bins created during pre-binning phase. Controls early-stage complexity. Default is 20.
bin_separator	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on changes in total Information Value. Default is 1e-6.
max_iterations	Integer. Maximum number of iterations for the optimization process. Default is 1000.
sketch_width	Integer. Width of the Count-Min Sketch (number of counters per hash function). Larger values reduce estimation error but increase memory usage. Must be >= 100. Default is 2000.
sketch_depth	Integer. Depth of the Count-Min Sketch (number of hash functions). Larger values reduce collision probability but increase computational overhead. Must be >= 3. Default is 5.

## Details

The Sketch-based algorithm follows these steps:

1. **Frequency Estimation:** Uses Count-Min Sketch to approximate the frequency of each category in a single data pass.

2. **Heavy Hitter Detection:** Identifies frequently occurring categories (above a threshold defined by `bin_cutoff`) using sketch estimates.
3. **Pre-binning:** Creates initial bins from detected heavy categories, grouping rare categories separately.
4. **Optimization:** Applies iterative merging based on statistical divergence measures to optimize Information Value (IV) while respecting bin count constraints (`min_bins`, `max_bins`).
5. **Monotonicity Enforcement:** Ensures the final binning has monotonic Weight of Evidence (WoE).

Key advantages of this approach:

- **Memory Efficiency:** Uses sub-linear space complexity, independent of dataset size.
- **Speed:** Single-pass algorithm with constant-time updates.
- **Scalability:** Suitable for streaming data or datasets too large to fit in memory.
- **Approximation:** Trades perfect accuracy for significant gains in speed and memory usage.

Mathematical concepts:

The Count-Min Sketch uses multiple hash functions to map items to counters:

$$CMS[i][h_i(x)]+ = 1 \quad \forall i \in \{1, \dots, d\}$$

where  $d$  is the sketch depth and  $w$  is the sketch width.

Frequency estimates are obtained by taking the minimum across all counters:

$$\hat{f}(x) = \min_i CMS[i][h_i(x)]$$

Statistical divergence between bins is measured using Jensen-Shannon divergence:

$$JSD(P||Q) = \frac{1}{2} [KL(P||M) + KL(Q||M)]$$

where  $M = \frac{1}{2}(P + Q)$  and  $KL$  is the Kullback-Leibler divergence.

Laplace smoothing is applied to WoE and IV calculations:

$$p_{smoothed} = \frac{count + \alpha}{total + 2\alpha}$$

## Value

A list containing the results of the optimal binning procedure:

- `id` Numeric vector of bin identifiers (1 to `n_bins`)
- `bin` Character vector of bin labels, which are combinations of original categories separated by `bin_separator`
- `woe` Numeric vector of Weight of Evidence values for each bin
- `iv` Numeric vector of Information Values for each bin
- `count` Integer vector of total observations in each bin

count\_pos Integer vector of positive outcomes in each bin  
 count\_neg Integer vector of negative outcomes in each bin  
 event\_rate Numeric vector of the observed event rate in each bin  
 total\_iv Numeric scalar. Total Information Value across all bins  
 converged Logical. Whether the algorithm converged  
 iterations Integer. Number of iterations performed

### Note

- Target variable must contain both 0 and 1 values.
- Due to the probabilistic nature of sketches, results may vary slightly between runs. For deterministic results, consider setting fixed random seeds in the underlying C++ code.
- Accuracy of frequency estimates depends on `sketch_width` and `sketch_depth`. Increase these parameters for higher precision at the cost of memory/computation.
- This algorithm is particularly beneficial when dealing with high-cardinality categorical features or streaming data scenarios.
- For small to medium datasets, deterministic algorithms like SBLP or MOB may provide more accurate results.

### References

Cormode, G., & Muthukrishnan, S. (2005). An improved data stream summary: the count-min sketch and its applications. *Journal of Algorithms*, 55(1), 58-75.

Lin, J., & Keogh, E., Wei, L., & Lonardi, S. (2007). Experiencing SAX: a novel symbolic representation of time series. *Data Mining and Knowledge Discovery*, 15(2), 107-144.

### Examples

```

# Generate sample data
set.seed(123)
n <- 10000
feature <- sample(letters, n, replace = TRUE, prob = c(rep(0.04, 13), rep(0.02, 13)))
# Create a relationship where early letters have higher probability
target_probs <- ifelse(as.numeric(factor(feature)) <= 10, 0.7, 0.3)
target <- rbinom(n, 1, prob = target_probs)

# Perform sketch-based optimal binning
result <- ob_categorical_sketch(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# With custom sketch parameters for higher accuracy
result_high_acc <- ob_categorical_sketch(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 7,
  sketch_width = 4000,
  sketch_depth = 7
)

```

```

)
# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 200)] <- NA
result_na <- ob_categorical_sketch(feature_with_na, target)

```

---

<b>ob_categorical_swb</b>	<i>Optimal Binning for Categorical Variables using Sliding Window Binning (SWB)</i>
---------------------------	---

---

## Description

This function performs optimal binning for categorical variables using the Sliding Window Binning (SWB) algorithm. This approach combines initial grouping based on frequency thresholds with iterative optimization to achieve monotonic Weight of Evidence (WoE) while maximizing Information Value (IV).

## Usage

```

ob_categorical_swb(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 20L,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000L
)

```

## Arguments

<b>feature</b>	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
<b>target</b>	An integer vector containing binary outcome values (0 or 1). Must be the same length as <b>feature</b> . Cannot contain missing values.
<b>min_bins</b>	Integer. Minimum number of bins to create. Must be at least 1. Default is 3.
<b>max_bins</b>	Integer. Maximum number of bins to create. Must be greater than or equal to <b>min_bins</b> . Default is 5.
<b>bin_cutoff</b>	Numeric. Minimum relative frequency threshold for individual categories. Categories with frequency below this proportion will be grouped together into a single "rare" bin. Value must be between 0 and 1. Default is 0.05 (5%).

max_n_prebins	Integer. Maximum number of initial bins created after the frequency-based grouping step. Used to control early-stage complexity. Default is 20.
bin_separator	Character string used to separate category names when multiple categories are merged into a single bin. Default is "%;%".
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on changes in total Information Value between iterations. Default is 1e-6.
max_iterations	Integer. Maximum number of iterations for the optimization process. Default is 1000.

## Details

The SWB algorithm follows these steps:

1. **Initialization:** Categories are initially grouped based on frequency thresholds (bin\_cutoff), separating frequent categories from rare ones.
2. **Preprocessing:** Initial bins are sorted by their WoE values to establish a baseline ordering.
3. **Sliding Window Optimization:** An iterative process evaluates adjacent bin pairs and merges those that contribute least to the overall Information Value or violate monotonicity constraints.
4. **Constraint Enforcement:** The final binning respects the specified min\_bins and max\_bins limits while maintaining WoE monotonicity.

Key features of this implementation:

- **Frequency-based Pre-grouping:** Automatically identifies and groups rare categories to reduce dimensionality.
- **Statistical Similarity Measures:** Utilizes Jensen-Shannon divergence to determine optimal merge candidates.
- **Monotonicity Preservation:** Ensures final bins exhibit consistent WoE trends (either increasing or decreasing).
- **Laplace Smoothing:** Employs additive smoothing to prevent numerical instabilities in WoE/IV calculations.

Mathematical concepts:

Weight of Evidence (WoE) with Laplace smoothing:

$$WoE = \ln \left( \frac{(p_{pos} + \alpha)/(N_{pos} + 2\alpha)}{(p_{neg} + \alpha)/(N_{neg} + 2\alpha)} \right)$$

Information Value (IV):

$$IV = \left( \frac{p_{pos} + \alpha}{N_{pos} + 2\alpha} - \frac{p_{neg} + \alpha}{N_{neg} + 2\alpha} \right) \times WoE$$

where  $p_{pos}$  and  $p_{neg}$  are bin-level counts,  $N_{pos}$  and  $N_{neg}$  are dataset-level totals, and  $\alpha$  is the smoothing parameter (default 0.5).

Jensen-Shannon Divergence between two bins:

$$JSD(P||Q) = \frac{1}{2} [KL(P||M) + KL(Q||M)]$$

where  $M = \frac{1}{2}(P + Q)$  and  $KL$  represents Kullback-Leibler divergence.

**Value**

A list containing the results of the optimal binning procedure:

```
id Numeric vector of bin identifiers (1 to n_bins)
bin Character vector of bin labels, which are combinations of original categories separated by
      bin_separator
woe Numeric vector of Weight of Evidence values for each bin
iv Numeric vector of Information Values for each bin
count Integer vector of total observations in each bin
count_pos Integer vector of positive outcomes in each bin
count_neg Integer vector of negative outcomes in each bin
event_rate Numeric vector of the observed event rate in each bin
total_iv Numeric scalar. Total Information Value across all bins
converged Logical. Whether the algorithm converged within specified tolerances
iterations Integer. Number of iterations performed
```

**Note**

- Target variable must contain both 0 and 1 values.
- The algorithm prioritizes monotonicity over strict adherence to bin count limits when conflicts arise.
- For datasets with very few unique categories (< 3), each category forms its own bin without optimization.
- Rare category grouping helps stabilize WoE estimates for infrequent values.

**Examples**

```
# Generate sample data with varying category frequencies
set.seed(456)
n <- 5000
# Create categories with power-law frequency distribution
categories <- c(
  rep("A", 1500), rep("B", 1000), rep("C", 800),
  rep("D", 500), rep("E", 300), rep("F", 200),
  sample(letters[7:26], 700, replace = TRUE)
)
feature <- sample(categories, n, replace = TRUE)
# Create target with dependency on top categories
target_probs <- ifelse(feature %in% c("A", "B"), 0.7,
  ifelse(feature %in% c("C", "D"), 0.5, 0.3))
target <- rbinom(n, 1, prob = target_probs)

# Perform sliding window binning
result <- ob_categorical_swb(feature, target)
print(result[c("bin", "woe", "iv", "count")])
```

```

# With stricter bin limits
result_strict <- ob_categorical_swb(
  feature = feature,
  target = target,
  min_bins = 4,
  max_bins = 6
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 100)] <- NA
result_na <- ob_categorical_swb(feature_with_na, target)

```

---

ob_categorical_udt	<i>Optimal Binning for Categorical Variables using a User-Defined Technique (UDT)</i>
--------------------	---

---

## Description

This function performs optimal binning for categorical variables using a User-Defined Technique (UDT) that combines frequency-based grouping with statistical similarity measures to create meaningful bins for predictive modeling.

## Usage

```

ob_categorical_udt(
  feature,
  target,
  min_bins = 3L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_preibins = 20L,
  bin_separator = "%;%",
  convergence_threshold = 1e-06,
  max_iterations = 1000L
)

```

## Arguments

feature	A character vector or factor representing the categorical predictor variable. Missing values (NA) will be converted to the string "NA" and treated as a separate category.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature. Cannot contain missing values.
min_bins	Integer. Minimum number of bins to create. Must be at least 1. Default is 3.

max_bins	Integer. Maximum number of bins to create. Must be greater than or equal to min_bins. Default is 5.
bin_cutoff	Numeric. Minimum relative frequency threshold for individual categories. Categories with frequency below this proportion will be merged into a collective "rare" bin before optimization. Value must be between 0 and 1. Default is 0.05 (5%).
max_n_prebins	Integer. Upper limit on initial bins after frequency filtering. Controls computational complexity in early stages. Default is 20.
bin_separator	Character string used to separate category names when multiple categories are combined into a single bin. Default is "%;%".
convergence_threshold	Numeric. Threshold for determining algorithm convergence based on relative changes in total Information Value. Default is 1e-6.
max_iterations	Integer. Maximum number of iterations permitted for the optimization routine. Default is 1000.

## Details

The UDT algorithm follows these steps:

1. **Initialization:** Each unique category is initially placed in its own bin.
2. **Frequency Filtering:** Categories below the bin\_cutoff frequency threshold are grouped into a single "rare" bin.
3. **Iterative Optimization:** Bins are progressively merged based on statistical similarity (measured by Jensen-Shannon divergence) until the desired number of bins (max\_bins) is achieved.
4. **Monotonicity Enforcement:** Final bins are sorted by Weight of Evidence to ensure consistent trends.

Key characteristics of this implementation:

- **Flexible Framework:** Designed as a customizable foundation for categorical binning approaches.
- **Statistical Rigor:** Uses information-theoretic measures to guide bin combination decisions.
- **Robust Estimation:** Implements Laplace smoothing to ensure stable WoE/IV calculations even with sparse data.
- **Efficiency Focus:** Employs targeted merging strategies to minimize computational overhead.

Mathematical foundations:

Laplace-smoothed probability estimates:

$$p_{smoothed} = \frac{count + \alpha}{total + 2\alpha}$$

Weight of Evidence calculation:

$$WoE = \ln \left( \frac{p_{pos,smoothed}}{p_{neg,smoothed}} \right)$$

Information Value computation:

$$IV = (p_{pos,smoothed} - p_{neg,smoothed}) \times WoE$$

Jensen-Shannon divergence between bins:

$$JSD(P||Q) = \frac{1}{2}[KL(P||M) + KL(Q||M)]$$

where  $M = \frac{1}{2}(P + Q)$  and  $KL$  denotes Kullback-Leibler divergence.

## Value

A list containing the results of the optimal binning procedure:

id Numeric vector of bin identifiers (1 to n\_bins)  
 bin Character vector of bin labels, which are combinations of original categories separated by bin\_separator  
 woe Numeric vector of Weight of Evidence values for each bin  
 iv Numeric vector of Information Values for each bin  
 count Integer vector of total observations in each bin  
 count\_pos Integer vector of positive outcomes in each bin  
 count\_neg Integer vector of negative outcomes in each bin  
 event\_rate Numeric vector of the observed event rate in each bin  
 total\_iv Numeric scalar. Total Information Value across all bins  
 converged Logical. Whether the algorithm converged  
 iterations Integer. Number of iterations executed

## Note

- Target variable must contain both 0 and 1 values.
- For datasets with 1 or 2 unique categories, no optimization occurs beyond basic WoE/IV calculation.
- The algorithm does not perform bin splitting; it only merges existing bins to respect max\_bins.
- Rare category pooling improves stability of WoE estimates for infrequent values.

## Examples

```

# Generate sample data with skewed category distribution
set.seed(789)
n <- 3000
# Power-law distributed categories
categories <- c(
  rep("X1", 1200), rep("X2", 800), rep("X3", 400),
  sample(LETTERS[4:20], 600, replace = TRUE)
)
feature <- sample(categories, n, replace = TRUE)

```

```

# Target probabilities based on category importance
probs <- ifelse(grepl("X", feature), 0.7,
                 ifelse(grepl("[A-C]", feature), 0.5, 0.3)
)
target <- rbinom(n, 1, prob = probs)

# Perform user-defined technique binning
result <- ob_categorical_udt(feature, target)
print(result[c("bin", "woe", "iv", "count")])

# Adjust parameters for finer control
result_custom <- ob_categorical_udt(
  feature = feature,
  target = target,
  min_bins = 2,
  max_bins = 7,
  bin_cutoff = 0.03
)

# Handling missing values
feature_with_na <- feature
feature_with_na[sample(length(feature_with_na), 150)] <- NA
result_na <- ob_categorical_udt(feature_with_na, target)

```

---

ob\_cutpoints\_cat*Binning Categorical Variables using Custom Cutpoints*

---

**Description**

This function applies user-defined binning to a categorical variable by grouping specified categories into bins and calculating Weight of Evidence (WoE) and Information Value (IV) for each bin.

**Usage**

```
ob_cutpoints_cat(feature, target, cutpoints)
```

**Arguments**

feature	A character vector or factor representing the categorical predictor variable.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature.
cutpoints	A character vector where each element defines a bin by concatenating the original category names with "+" as separator.

## Details

The function takes a character vector defining how categories should be grouped. Each element in the `cutpoints` vector defines one bin by listing the original categories that should be merged, separated by "+" signs.

For example, if you want to create two bins from categories "A", "B", "C", "D":

- Bin 1: "A+B"
- Bin 2: "C+D"

## Value

A list containing:

`woefeature` Numeric vector of WoE values corresponding to each observation in the input feature

`woebin` Data frame with one row per bin containing:

- `bin`: The bin definition (original categories joined by "+")
- `count`: Total number of observations in the bin
- `count_pos`: Number of positive outcomes (`target=1`) in the bin
- `count_neg`: Number of negative outcomes (`target=0`) in the bin
- `woe`: Weight of Evidence for the bin
- `iv`: Information Value contribution of the bin

## Note

- Target variable must contain only 0 and 1 values.
- Every unique category in `feature` must be included in exactly one bin definition in `cutpoints`.
- Categories not mentioned in `cutpoints` will be assigned to bin 0 (which may lead to unexpected results).

## Examples

```
# Sample data
feature <- c("A", "B", "C", "D", "A", "B", "C", "D")
target <- c(1, 0, 1, 0, 1, 1, 0, 0)

# Define custom bins: (A,B) and (C,D)
cutpoints <- c("A+B", "C+D")

# Apply binning
result <- ob_cutpoints_cat(feature, target, cutpoints)

# View bin statistics
print(result$woebin)

# View WoE-transformed feature
print(result$woefeature)
```

## Description

This function applies user-defined binning to a numerical variable by using specified cutpoints to create intervals and calculates Weight of Evidence (WoE) and Information Value (IV) for each interval bin.

## Usage

```
ob_cutpoints_num(feature, target, cutpoints)
```

## Arguments

feature	A numeric vector representing the continuous predictor variable.
target	An integer vector containing binary outcome values (0 or 1). Must be the same length as feature.
cutpoints	A numeric vector of cutpoints that define bin boundaries. These will be automatically sorted in ascending order.

## Details

The function takes a numeric vector of cutpoints that define the boundaries between bins. For  $n$  cutpoints,  $n+1$  bins are created:

- Bin 1:  $(-\infty, cutpoint_1)$
- Bin 2:  $[cutpoint_1, cutpoint_2)$
- ...
- Bin  $n+1$ :  $[cutpoint_n, +\infty)$

## Value

A list containing:

woefeature Numeric vector of WoE values corresponding to each observation in the input feature  
 woebin Data frame with one row per bin containing:

- bin: The bin interval notation (e.g., "[10.00;20.00)")
- count: Total number of observations in the bin
- count\_pos: Number of positive outcomes (target=1) in the bin
- count\_neg: Number of negative outcomes (target=0) in the bin
- woe: Weight of Evidence for the bin
- iv: Information Value contribution of the bin

**Note**

- Target variable must contain only 0 and 1 values.
- Cutpoints are sorted automatically in ascending order.
- Interval notation uses "[" for inclusive and ")" for exclusive bounds.
- Infinite values in feature are handled appropriately.

**Examples**

```
# Sample data
feature <- c(5, 15, 25, 35, 45, 55, 65, 75)
target <- c(0, 0, 1, 1, 1, 1, 0, 0)

# Define custom cutpoints
cutpoints <- c(30, 60)

# Apply binning
result <- ob_cutpoints_num(feature, target, cutpoints)

# View bin statistics
print(result$woebin)

# View WoE-transformed feature
print(result$woefeature)
```

---

ob\_gains\_table*Compute Comprehensive Gains Table from Binning Results*

---

**Description**

This function serves as a high-performance engine (implemented in C++) to calculate a comprehensive set of credit scoring and classification metrics based on pre-aggregated binning results. It takes a list of bin counts and computes metrics such as Information Value (IV), Weight of Evidence (WoE), Kolmogorov-Smirnov (KS), Gini, Lift, and various entropy-based divergence measures.

**Usage**

```
ob_gains_table(binning_result)
```

**Arguments**

**binning\_result** A named list or `data.frame` containing the following atomic vectors (all must have the same length):

- id** Numeric vector of bin identifiers. Determines the sort order for cumulative metrics (e.g., KS, Recall).
- bin** Character vector of bin labels/intervals.
- count** Numeric vector of total observations per bin ( $O_i$ ).
- count\_pos** Numeric vector of positive (event) counts per bin ( $E_i$ ).
- count\_neg** Numeric vector of negative (non-event) counts per bin ( $NE_i$ ).

## Details

### Mathematical Definitions:

Let  $E_i$  and  $NE_i$  be the number of events and non-events in bin  $i$ , and  $E_{total}$ ,  $NE_{total}$  be the population totals.

### Weight of Evidence (WoE) & Information Value (IV):

$$WoE_i = \ln \left( \frac{E_i/E_{total}}{NE_i/NE_{total}} \right)$$

$$IV_i = \left( \frac{E_i}{E_{total}} - \frac{NE_i}{NE_{total}} \right) \times WoE_i$$

### Kolmogorov-Smirnov (KS):

$$KS_i = \left| \sum_{j=1}^i \frac{E_j}{E_{total}} - \sum_{j=1}^i \frac{NE_j}{NE_{total}} \right|$$

### Lift:

$$Lift_i = \frac{E_i/(E_i + NE_i)}{E_{total}/(E_{total} + NE_{total})}$$

**Kullback-Leibler Divergence (Bernoulli):** Measures the divergence between the bin's event rate  $p_i$  and the global event rate  $p_{global}$ :

$$KL_i = p_i \ln \left( \frac{p_i}{p_{global}} \right) + (1 - p_i) \ln \left( \frac{1 - p_i}{1 - p_{global}} \right)$$

## Value

A data.frame with the following columns (metrics calculated per bin):

**Identifiers** id, bin

**Counts & Rates** count, pos, neg, pos\_rate ( $\pi_i$ ), neg\_rate ( $1 - \pi_i$ ), count\_perc ( $O_i/O_{total}$ )

**Distributions (Shares)** pos\_perc ( $D_1(i)$ ): Share of Bad), neg\_perc ( $D_0(i)$ ): Share of Good)

**Cumulative Statistics** cum\_pos, cum\_neg, cum\_pos\_perc ( $CDF_1$ ), cum\_neg\_perc ( $CDF_0$ ), cum\_count\_perc

**Credit Scoring Metrics** woe, iv, total\_iv, ks, lift, odds\_pos, odds\_ratio

**Advanced Metrics** gini\_contribution, log\_likelihood, kl\_divergence, js\_divergence

**Classification Metrics** precision, recall, f1\_score

## Examples

```
# Manually constructed binning result
bin_res <- list(
  id = 1:3,
  bin = c("Low", "Medium", "High"),
  count = c(100, 200, 50),
  count_pos = c(5, 30, 20),
  count_neg = c(95, 170, 30)
```

```

)
gt <- ob_gains_table(bin_res)
print(gt[, c("bin", "woe", "iv", "ks")])

```

---

### ob\_gains\_table\_feature

*Compute Gains Table for a Binned Feature Vector*

---

## Description

Calculates a full gains table by aggregating a raw binned dataframe against a binary target. Unlike [ob\\_gains\\_table](#) which expects pre-aggregated counts, this function takes observation-level data, aggregates it by the specified group variable (bin, WoE, or ID), and then computes all statistical metrics.

## Usage

```
ob_gains_table_feature(binned_df, target, group_var = "bin")
```

## Arguments

binned_df	A <code>data.frame</code> resulting from a binning transformation (e.g., via <code>obwoe_apply</code> ), containing at least the following columns:
	feature Original feature values (optional, for reference).
	bin Character vector of bin labels.
	woe Numeric vector of Weight of Evidence values.
	idbin Numeric vector of bin IDs (required for correct sorting).
target	A numeric vector of binary outcomes (0 for non-event, 1 for event). Must have the same length as <code>binned_df</code> . Missing values are not allowed.
group_var	Character string specifying the aggregation key. Options: <ul style="list-style-type: none"> <li>• "bin": Group by bin label (default).</li> <li>• "woe": Group by WoE value.</li> <li>• "idbin": Group by bin ID.</li> </ul>

## Details

**Aggregation and Sorting:** The function first aggregates the binary target by the specified `group_var`. Crucially, it uses the `idbin` column to sort the resulting groups. This ensures that cumulative metrics (like KS and Gini) are calculated based on the logical order of the bins (e.g., low score to high score), not alphabetical order.

**Advanced Metrics:** In addition to standard credit scoring metrics, this function computes:

- **Jensen-Shannon Divergence:** A symmetrized and smoothed version of KL divergence, useful for measuring stability between the bin distribution and the population distribution.
- **F1-Score, Precision, Recall:** Treating each bin as a potential classification threshold.

### Value

A `data.frame` containing the same extensive set of metrics as `ob_gains_table`, aggregated by `group_var` and sorted by `idbin`.

### References

Siddiqi, N. (2006). *Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring*. Wiley.

Kullback, S., & Leibler, R. A. (1951). On Information and Sufficiency. *The Annals of Mathematical Statistics*.

### Examples

```
# Mock data representing a binned feature
df_binned <- data.frame(
  feature = c(10, 20, 30, 10, 20, 50),
  bin = c("Low", "Mid", "High", "Low", "Mid", "High"),
  woe = c(-0.5, 0.2, 1.1, -0.5, 0.2, 1.1),
  idbin = c(1, 2, 3, 1, 2, 3)
)
target <- c(0, 0, 1, 1, 0, 1)

# Calculate gains table grouped by bin ID
gt <- ob_gains_table_feature(df_binned, target, group_var = "idbin")

# Inspect key metrics
print(gt[, c("id", "count", "pos_rate", "lift", "js_divergence")])
```

---

ob\_numerical\_bb

*Optimal Binning for Numerical Variables using Branch and Bound Algorithm*

---

### Description

Performs supervised discretization of continuous numerical variables using a Branch and Bound-style approach. This algorithm optimally creates bins based on the relationship with a binary target variable, maximizing Information Value (IV) while optionally enforcing monotonicity in Weight of Evidence (WoE).

### Usage

```
ob_numerical_bb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
```

```

  bin_cutoff = 0.05,
  max_n_prebins = 20,
  is_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

feature	A numeric vector representing the continuous predictor variable to be binned. NA values are handled by exclusion during the pre-binning phase.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ will be merged with neighbors. Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. The number of initial quantiles to generate during the pre-binning phase. Higher values provide more granular starting points but increase computation time. Must be $\geq \text{min\_bins}$ . Defaults to 20.
is_monotonic	Logical. If TRUE, the algorithm enforces a strict monotonic relationship (increasing or decreasing) between the bin indices and their WoE values. This makes the variable more interpretable for linear models. Defaults to TRUE.
convergence_threshold	Numeric. The threshold for the change in total IV to determine convergence during the iterative merging process. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.

## Details

The algorithm proceeds in several distinct phases to ensure stability and optimality:

- 1. Pre-binning:** The numerical feature is initially discretized into `max_n_prebins` using quantiles. This handles outliers and provides a granular starting point.
- 2. Rare Bin Management:** Bins containing fewer observations than the threshold defined by `bin_cutoff` are iteratively merged with their nearest neighbors to ensure statistical robustness.
- 3. Monotonicity Enforcement (Optional):** If `is_monotonic` = TRUE, the algorithm checks if the WoE trend is strictly increasing or decreasing. If not, it simulates merges in both directions to find the path that preserves the maximum possible Information Value while satisfying the monotonicity constraint.
- 4. Optimization Phase:** The algorithm iteratively merges adjacent bins that have the lowest contribution to the total Information Value (IV). This process continues until the number of bins is reduced to `max_bins` or the change in IV falls below `convergence_threshold`.

### Information Value (IV) Interpretation:

- $< 0.02$ : Not predictive
- $0.02$  to  $0.1$ : Weak predictive power
- $0.1$  to  $0.3$ : Medium predictive power
- $0.3$  to  $0.5$ : Strong predictive power
- $> 0.5$ : Suspiciously high (check for leakage)

### Value

A list containing the binning results:

- **id**: Integer vector of bin identifiers (1 to k).
- **bin**: Character vector of bin labels in interval notation (e.g., "(0.5;1.2]").
- **woe**: Numeric vector of Weight of Evidence for each bin.
- **iv**: Numeric vector of Information Value contribution per bin.
- **count**: Integer vector of total observations per bin.
- **count\_pos**: Integer vector of positive cases (target=1) per bin.
- **count\_neg**: Integer vector of negative cases (target=0) per bin.
- **cutpoints**: Numeric vector of upper boundaries for the bins (excluding Inf).
- **converged**: Logical indicating if the algorithm converged properly.
- **iterations**: Integer count of iterations performed.
- **total\_iv**: The total Information Value of the binned variable.

### Examples

```
# Example: Binning a variable with a sigmoid relationship to target
set.seed(123)
n <- 1000
# Generate feature
feature <- rnorm(n)

# Generate target based on logistic probability
prob <- 1 / (1 + exp(-2 * feature))
target <- rbinom(n, 1, prob)

# Perform Optimal Binning
result <- ob_numerical_bb(feature, target,
  min_bins = 3,
  max_bins = 5,
  is_monotonic = TRUE
)

# Check results
print(data.frame(
  Bin = result$bin,
  Count = result$count,
```

```

WoE = round(result$woe, 4),
IV = round(result$iv, 4)
))

cat("Total IV:", result$total_iv, "\n")

```

---

ob\_numerical\_cm

*Optimal Binning for Numerical Variables using Enhanced ChiMerge Algorithm*

---

## Description

Performs supervised discretization of continuous numerical variables using the ChiMerge algorithm (Kerber, 1992) or the Chi2 algorithm (Liu & Setiono, 1995). This function merges adjacent bins based on Chi-square statistics to maximize the discrimination of the binary target variable while ensuring monotonicity and statistical robustness.

## Usage

```

ob_numerical_cm(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  init_method = "equal_frequency",
  chi_merge_threshold = 0.05,
  use_chi2_algorithm = FALSE
)

```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) are not supported and should be handled before binning.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ will be merged. Value must be in (0, 1). Defaults to 0.05.

`max_n_prebins` Integer. The number of initial bins created during the pre-binning phase before the merging process begins. Higher values provide more granular starting points. Must be  $\geq \text{max\_bins}$ . Defaults to 20.

`convergence_threshold` Numeric. The threshold for the change in total IV to determine convergence during the iterative merging process. Defaults to 1e-6.

`max_iterations` Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.

`init_method` Character string specifying the initialization method. Options are "equal\_frequency" (quantile-based) or "equal\_width". Defaults to "equal\_frequency".

`chi_merge_threshold` Numeric. The significance level ( $\alpha$ ) for the Chi-square test. Pairs of bins with a p-value  $> \text{chi\_merge\_threshold}$  are candidates for merging. Defaults to 0.05.

`use_chi2_algorithm` Logical. If TRUE, uses the Chi2 algorithm variant which performs multi-phase merging with decreasing significance levels (0.5, 0.1, 0.05, 0.01, ...). This is often more robust for noisy data. Defaults to FALSE.

## Details

The function implements two major discretization strategies:

### 1. Standard ChiMerge:

- Initializes bins using `init_method`.
- Iteratively merges adjacent bins with the lowest  $\chi^2$  statistic.
- Merging continues until all adjacent pairs have a p-value less than `chi_merge_threshold` or the number of bins reaches `max_bins`.

### 2. Chi2 Algorithm:

- Activated when `use_chi2_algorithm` = TRUE.
- Performs multiple passes with decreasing significance levels (0.5  $\rightarrow$  0.001) to automatically select the optimal significance threshold.
- Checks for inconsistency rates in the data during the process.

Both methods include post-processing steps to enforce:

- **Minimum Bin Size:** Merging rare bins smaller than `bin_cutoff`.
- **Monotonicity:** Ensuring WoE trend is strictly increasing or decreasing to improve model interpretability.

## Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers (1 to k).
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.

- iv: Numeric vector of Information Value contribution per bin.
- count: Integer vector of total observations per bin.
- count\_pos: Integer vector of positive cases (target=1).
- count\_neg: Integer vector of negative cases (target=0).
- cutpoints: Numeric vector of upper boundaries (excluding Inf).
- converged: Logical indicating if the algorithm converged.
- iterations: Integer count of iterations performed.
- total\_iv: The total Information Value of the binned variable.
- algorithm: String identifying the algorithm used ("ChiMerge" or "Chi2").
- monotonic: Logical indicating if the final WoE trend is monotonic.

## References

Kerber, R. (1992). ChiMerge: Discretization of numeric attributes. *Proceedings of the Tenth National Conference on Artificial Intelligence*, 123-128.

Liu, H., & Setiono, R. (1995). Chi2: Feature selection and discretization of numeric attributes. *Tools with Artificial Intelligence*, 388-391.

## Examples

```
# Example 1: Standard ChiMerge
set.seed(123)
feature <- rnorm(1000)
# Create a target with a relationship to the feature
target <- rbinom(1000, 1, plogis(2 * feature))

res_cm <- ob_numerical_cm(feature, target,
  min_bins = 3,
  max_bins = 6,
  init_method = "equal_frequency"
)

print(res_cm$bin)
print(res_cm$iv)

# Example 2: Using the Chi2 Algorithm variant
res_chi2 <- ob_numerical_cm(feature, target,
  min_bins = 3,
  max_bins = 6,
  use_chi2_algorithm = TRUE
)

cat("Total IV (ChiMerge):", res_cm$total_iv, "\n")
cat("Total IV (Chi2):", res_chi2$total_iv, "\n")
```

## Description

Performs supervised discretization of continuous numerical variables using the theoretical framework proposed by Zeng (2013). This method creates bins that maximize a specified divergence measure (e.g., Kullback-Leibler, Hellinger) between the distributions of positive and negative cases, effectively maximizing the Information Value (IV) or other discriminatory statistics.

## Usage

```
ob_numerical_dmv(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  is_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  bin_method = c("woe1", "woe"),
  divergence_method = c("l2", "he", "kl", "tr", "klj", "sc", "js", "l1", "ln")
)
```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) are excluded during the pre-binning phase.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ will be merged. Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. The number of initial quantiles to generate during the pre-binning phase. Defaults to 20.
is_monotonic	Logical. If TRUE, the algorithm enforces a strict monotonic relationship (increasing or decreasing) between the bin indices and their WoE values. Defaults to TRUE.
convergence_threshold	Numeric. The threshold for the change in total divergence to determine convergence during the iterative merging process. Defaults to 1e-6.

max_iterations	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.
bin_method	Character string specifying the formula for Weight of Evidence calculation: <ul style="list-style-type: none"> <li>• "woe": Standard definition <math>\ln((p_i/P)/(n_i/N))</math>.</li> <li>• "woe1": Zeng's definition <math>\ln(p_i/n_i)</math> (direct log odds).</li> </ul> Defaults to "woe1".
divergence_method	Character string specifying the divergence measure to maximize. Available options: <ul style="list-style-type: none"> <li>• "iv": Information Value (conceptually similar to KL).</li> <li>• "he": Hellinger Distance.</li> <li>• "k1": Kullback-Leibler Divergence.</li> <li>• "tr": Triangular Discrimination.</li> <li>• "k1j": Jeffrey's Divergence (Symmetric KL).</li> <li>• "sc": Symmetric Chi-Square Divergence.</li> <li>• "js": Jensen-Shannon Divergence.</li> <li>• "l1": Manhattan Distance (L1 Norm).</li> <li>• "l2": Euclidean Distance (L2 Norm).</li> <li>• "ln": Chebyshev Distance (L-infinity Norm).</li> </ul> Defaults to "l2".

## Details

This algorithm implements the "Metric Divergence Measures" framework. Unlike standard ChiMerge which uses statistical significance, this method uses a branch-and-bound approach to minimize the loss of a specific divergence metric when merging bins.

### The Process:

1. **Pre-binning:** Generates granular bins based on quantiles.
2. **Rare Merging:** Merges bins smaller than `bin_cutoff`.
3. **Monotonicity:** If `is_monotonic` = TRUE, forces the WoE trend to be monotonic by merging "violating" bins in the direction that maximizes the total divergence.
4. **Optimization:** Iteratively merges the pair of adjacent bins that results in the smallest loss of total divergence, until `max_bins` is reached.

## Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `divergence`: Numeric vector of the chosen divergence contribution per bin.
- `count`: Integer vector of total observations per bin.

- count\_pos: Integer vector of positive cases.
- count\_neg: Integer vector of negative cases.
- cutpoints: Numeric vector of upper boundaries (excluding Inf).
- total\_divergence: The sum of the divergence measure across all bins.
- bin\_method: The WoE calculation method used.
- divergence\_method: The divergence measure used.

## References

Zeng, G. (2013). Metric Divergence Measures and Information Value in Credit Scoring. *Journal of the Operational Research Society*, 64(5), 712-731.

## Examples

```
# Example using the "he" (Hellinger) distance
set.seed(123)
feature <- rnorm(1000)
target <- rbinom(1000, 1, plogis(feature))

result <- ob_numerical_dmiv(feature, target,
  min_bins = 3,
  max_bins = 5,
  divergence_method = "he",
  bin_method = "woe"
)

print(result$bin)
print(result$divergence)
print(paste("Total Hellinger Distance:", round(result$total_divergence, 4)))
```

## Description

Performs supervised discretization of continuous numerical variables using a greedy heuristic approach that resembles Dynamic Programming. This method is particularly effective at strictly enforcing monotonic trends (ascending or descending) in the Weight of Evidence (WoE), which is critical for the interpretability of logistic regression models in credit scoring.

**Usage**

```
ob_numerical_dp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  monotonic_trend = c("auto", "ascending", "descending", "none")
)
```

**Arguments**

<code>feature</code>	A numeric vector representing the continuous predictor variable. Missing values (NA) should be handled prior to binning, as they are not supported by this algorithm.
<code>target</code>	An integer vector of binary outcomes (0/1) corresponding to each observation in <code>feature</code> . Must have the same length as <code>feature</code> .
<code>min_bins</code>	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
<code>max_bins</code>	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
<code>bin_cutoff</code>	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ will be merged. Value must be in (0, 1). Defaults to 0.05.
<code>max_n_prebins</code>	Integer. The number of initial quantiles to generate during the pre-binning phase. Defaults to 20.
<code>convergence_threshold</code>	Numeric. The threshold for the change in metrics to determine convergence during the iterative merging process. Defaults to 1e-6.
<code>max_iterations</code>	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.
<code>monotonic_trend</code>	Character string specifying the desired direction of the Weight of Evidence (WoE) trend. <ul style="list-style-type: none"> <li>• "auto": Automatically determines the most likely trend (ascending or descending) based on the correlation between the feature and the target.</li> <li>• "ascending": Forces the WoE to increase as the feature value increases.</li> <li>• "descending": Forces the WoE to decrease as the feature value increases.</li> <li>• "none": Does not enforce any monotonic constraint (allows peaks and valleys).</li> </ul> Defaults to "auto".

## Details

Although named "DP" (Dynamic Programming) in some contexts, this implementation primarily uses a **greedy heuristic** to optimize the Information Value (IV) while satisfying constraints.

### Algorithm Steps:

1. **Pre-binning:** Generates initial granular bins based on quantiles.
2. **Trend Determination:** If `monotonic_trend` = "auto", calculates the Pearson correlation between the feature and target to decide if the WoE should increase or decrease.
3. **Monotonicity Enforcement:** Iteratively merges adjacent bins that violate the determined or requested trend.
4. **Constraint Satisfaction:** Merges rare bins (below `bin_cutoff`) and ensures the number of bins is within `[min_bins, max_bins]`.
5. **Optimization:** Greedily merges similar bins (based on WoE difference) to reduce complexity while attempting to preserve information.

This method is often preferred when strict business logic dictates a specific relationship direction (e.g., "higher income must imply lower risk").

## Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `iv`: Numeric vector of Information Value contribution per bin.
- `count`: Integer vector of total observations per bin.
- `count_pos`: Integer vector of positive cases.
- `count_neg`: Integer vector of negative cases.
- `event_rate`: Numeric vector of the target event rate in each bin.
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `total_iv`: The total Information Value of the binned variable.
- `monotonic_trend`: The actual trend enforced ("ascending", "descending", or "none").
- `execution_time_ms`: Execution time in milliseconds.

## See Also

[ob\\_numerical\\_cm](#), [ob\\_numerical\\_bb](#)

## Examples

```

# Example: forcing a descending trend
set.seed(123)
feature <- runif(1000, 0, 100)
# Target has a complex relationship, but we want to force a linear view
target <- rbinom(1000, 1, 0.5 + 0.003 * feature) # slightly positive trend

# Force "descending" (even if data suggests ascending) to see enforcement
result <- ob_numerical_dp(feature, target,
  min_bins = 3,
  max_bins = 5,
  monotonic_trend = "descending"
)

print(result$bin)
print(result$woe) # Should be strictly decreasing

```

---

ob\_numerical\_ewb

*Hybrid Optimal Binning using Equal-Width Initialization and IV Optimization*

---

## Description

Performs supervised discretization of continuous numerical variables using a hybrid approach. The algorithm initializes with an Equal-Width Binning (EWB) strategy to capture the scale of the variable, followed by an iterative, supervised optimization phase that merges bins to maximize Information Value (IV) and enforce monotonicity.

## Usage

```

ob_numerical_ewb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  is_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) are excluded during the pre-binning phase but should ideally be handled prior to binning.
---------	--

target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ are merged with their most similar neighbor (based on event rate). Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. The number of initial equal-width intervals to generate during the pre-binning phase. This parameter defines the initial granularity/search space. Defaults to 20.
is_monotonic	Logical. If TRUE, the algorithm enforces a strict monotonic relationship (increasing or decreasing) between the bin indices and their Weight of Evidence (WoE). Defaults to TRUE.
convergence_threshold	Numeric. The threshold for determining convergence during the iterative merging process. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.

## Details

Unlike standard Equal-Width binning which is purely unsupervised, this function implements a **Hybrid Discretization Pipeline**:

1. **Phase 1: Unsupervised Initialization (Scale Preservation)** The range of the feature  $[min(x), max(x)]$  is divided into `max_n_prebins` intervals of equal width  $w = (max(x) - min(x))/N$ . This step preserves the cardinal magnitude of the data but is sensitive to outliers.
2. **Phase 2: Statistical Stabilization** Bins falling below the `bin_cutoff` threshold are merged. Unlike naive approaches, this implementation merges rare bins with the neighbor that has the most similar class distribution (event rate), minimizing the distortion of the predictive relationship.
3. **Phase 3: Monotonicity Enforcement** If `is_monotonic` = TRUE, the algorithm checks for non-monotonic trends in the Weight of Evidence (WoE). Violating adjacent bins are iteratively merged to ensure a strictly increasing or decreasing relationship, which is a key requirement for interpretable Logistic Regression scorecards.
4. **Phase 4: IV-Based Optimization** If the number of bins exceeds `max_bins`, the algorithm applies a hierarchical bottom-up merging strategy. It calculates the *Information Value Loss* for every possible pair of adjacent bins:

$$\Delta IV = (IV_i + IV_{i+1}) - IV_{merged}$$

The pair minimizing this loss is merged, ensuring that the final coarse classes retain the maximum possible predictive power of the original variable.

**Technical Note on Outliers:** Because the initialization is based on the range, extreme outliers can compress the majority of the data into a single initial bin. If your data is highly skewed or contains outliers, consider using [ob\\_numerical\\_cm](#) (Quantile/ChiMerge) or winsorizing the data before using this function.

## Value

A list containing the binning results:

- **id**: Integer vector of bin identifiers.
- **bin**: Character vector of bin labels in interval notation.
- **woe**: Numeric vector of Weight of Evidence for each bin.
- **iv**: Numeric vector of Information Value contribution per bin.
- **count**: Integer vector of total observations per bin.
- **count\_pos**: Integer vector of positive cases.
- **count\_neg**: Integer vector of negative cases.
- **cutpoints**: Numeric vector of upper boundaries (excluding Inf).
- **total\_iv**: The total Information Value of the binned variable.
- **converged**: Logical indicating if the algorithm converged.

## References

Dougherty, J., Kohavi, R., & Sahami, M. (1995). Supervised and unsupervised discretization of continuous features. *Machine Learning Proceedings*, 194-202.

Siddiqi, N. (2012). *Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring*. John Wiley & Sons.

Catlett, J. (1991). On changing continuous attributes into ordered discrete attributes. *Proceedings of the European Working Session on Learning on Machine Learning*, 164-178.

## See Also

[ob\\_numerical\\_cm](#) for Quantile/Chi-Square binning, [ob\\_numerical\\_dp](#) for Dynamic Programming approaches.

## Examples

```
# Example 1: Uniform distribution (Ideal for Equal-Width)
set.seed(123)
feature <- runif(1000, 0, 100)
target <- rbinom(1000, 1, plogis(0.05 * feature - 2))

res_ewb <- ob_numerical_ewb(feature, target, max_bins = 5)
print(res_ewb$bin)
print(paste("Total IV:", round(res_ewb$total_iv, 4)))

# Example 2: Effect of Outliers (The weakness of Equal-Width)
feature_outlier <- c(feature, 10000) # One extreme outlier
target_outlier <- c(target, 0)

# Note: The algorithm tries to recover, but the initial split is distorted
res_outlier <- ob_numerical_ewb(feature_outlier, target_outlier, max_bins = 5)
print(res_outlier$bin)
```

---

ob\_numerical\_fast\_mdlp*Optimal Binning using MDLP with Monotonicity Constraints*

---

**Description**

Performs supervised discretization of continuous numerical variables using the Minimum Description Length Principle (MDLP) algorithm, enhanced with optional monotonicity constraints on the Weight of Evidence (WoE). This method is particularly suitable for creating interpretable bins for logistic regression models in domains like credit scoring.

**Usage**

```
ob_numerical_fast_mdlp(
  feature,
  target,
  min_bins = 2L,
  max_bins = 5L,
  bin_cutoff = 0.05,
  max_n_prebins = 100L,
  convergence_threshold = 1e-06,
  max_iterations = 1000L,
  force_monotonicity = TRUE
)
```

**Arguments**

<code>feature</code>	A numeric vector representing the continuous predictor variable. Missing values (NA) are excluded during the binning process.
<code>target</code>	An integer vector of binary outcomes (0/1) corresponding to each observation in <code>feature</code> . Must have the same length as <code>feature</code> .
<code>min_bins</code>	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 2.
<code>max_bins</code>	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
<code>bin_cutoff</code>	Numeric. Currently unused in this implementation (reserved for future versions). Defaults to 0.05.
<code>max_n_prebins</code>	Integer. Currently unused in this implementation (reserved for future versions). Defaults to 100.
<code>convergence_threshold</code>	Numeric. The threshold for determining convergence during the iterative monotonicity enforcement process. Defaults to 1e-6.
<code>max_iterations</code>	Integer. Safety limit for the maximum number of iterations in the monotonicity enforcement phase. Defaults to 1000.

#### force\_monotonicity

Logical. If TRUE, the algorithm enforces a strict monotonic relationship (increasing or decreasing) between the bin indices and their Weight of Evidence (WoE) values. Defaults to TRUE.

#### Details

This function implements a sophisticated hybrid approach combining the classic MDLP algorithm with modern monotonicity constraints.

#### Algorithm Pipeline:

1. **Data Preparation:** Removes NA values and sorts the data by feature value.

2. **MDLP Discretization (Fayyad & Irani, 1993):**

- Recursively evaluates all possible binary splits of the sorted data.
- For each potential split, calculates the Information Gain (IG).
- Applies the MDLP stopping criterion:

$$IG > \frac{\log_2(N - 1) + \Delta}{N}$$

where  $N$  is the total number of samples and  $\Delta = \log_2(3^k - 2) - k \cdot E(S)$  (for binary classification,  $k = 2$ ).

- Only accepts splits that significantly reduce entropy beyond what would be expected by chance, balancing model fit with complexity.

3. **Constraint Enforcement:**

- **Min/Max Bins:** Adjusts the number of bins to meet [min\_bins, max\_bins] requirements through intelligent splitting or merging.
- **Monotonicity (if enabled):** Iteratively merges adjacent bins with the most similar WoE values until a strictly increasing or decreasing trend is achieved across all bins.

#### Technical Notes:

- The algorithm uses Laplace smoothing ( $\alpha = 0.5$ ) when calculating WoE to prevent  $\log(0)$  errors for bins with pure class distributions.
- When all feature values are identical, the algorithm creates artificial bins.
- The monotonicity enforcement phase is iterative and uses the convergence\_threshold to determine when changes in WoE become negligible.

#### Value

A list containing the binning results:

- **id:** Integer vector of bin identifiers.
- **bin:** Character vector of bin labels in interval notation.
- **woe:** Numeric vector of Weight of Evidence for each bin.
- **iv:** Numeric vector of Information Value contribution per bin.
- **count:** Integer vector of total observations per bin.

- count\_pos: Integer vector of positive cases.
- count\_neg: Integer vector of negative cases.
- cutpoints: Numeric vector of upper boundaries (excluding Inf).
- converged: Logical indicating if the monotonicity enforcement converged.
- iterations: Integer count of iterations in monotonicity phase.

## References

Fayyad, U. M., & Irani, K. B. (1993). Multi-interval discretization of continuous-valued attributes for classification learning. *Proceedings of the 13th International Joint Conference on Artificial Intelligence*, 1022-1029.

Kurgan, L. A., & Musilek, P. (2006). A survey of techniques. *IEEE Transactions on Knowledge and Data Engineering*, 18(5), 673-689.

Garcia, S., Luengo, J., & Herrera, F. (2013). Data preprocessing in data mining. *Springer Science & Business Media*.

## See Also

[ob\\_numerical\\_cm](#) for ChiMerge-based approaches, [ob\\_numerical\\_dp](#) for dynamic programming methods.

## Examples

```
# Example: Standard usage with monotonicity
set.seed(123)
feature <- rnorm(1000)
target <- rbinom(1000, 1, plogis(2 * feature)) # Positive relationship

result <- ob_numerical_fast_mdlp(feature, target,
  min_bins = 3,
  max_bins = 6,
  force_monotonicity = TRUE
)

print(result$bin)
print(result$woe) # Should show a monotonic trend

# Example: Disabling monotonicity for exploratory analysis
result_no_mono <- ob_numerical_fast_mdlp(feature, target,
  min_bins = 3,
  max_bins = 6,
  force_monotonicity = FALSE
)

print(result_no_mono$woe) # May show non-monotonic patterns
```

---

ob_numerical_fetb	<i>Optimal Binning using Fisher's Exact Test</i>
-------------------	--

---

## Description

Performs supervised discretization of continuous numerical variables using Fisher's Exact Test. This method iteratively merges adjacent bins that are statistically similar (highest p-value) while strictly enforcing a monotonic Weight of Evidence (WoE) trend.

## Usage

```
ob_numerical_fetb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) should be handled prior to binning.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
max_n_prebins	Integer. The number of initial quantiles to generate during the pre-binning phase. Defaults to 20.
convergence_threshold	Numeric. The threshold for the change in Information Value (IV) to determine convergence during the iterative merging process. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.

## Details

The **Fisher's Exact Test Binning (FETB)** algorithm provides a robust statistical alternative to ChiMerge.

### Key Differences from ChiMerge:

- **Exact Probability:** Instead of relying on the Chi-Square asymptotic approximation (which can be unreliable for small bin counts), FETB calculates the exact hypergeometric probability of independence between the bin index and the target.
- **Merge Criterion:** In each step, the algorithm identifies the pair of adjacent bins with the *highest* p-value (indicating they are the most statistically indistinguishable) and merges them.
- **Monotonicity:** The algorithm incorporates a check after every merge to ensure the WoE trend remains monotonic, merging strictly violating bins immediately.

This method is particularly recommended when working with smaller datasets or highly imbalanced target classes, where the assumptions of the Chi-Square test might be violated.

### Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `iv`: Numeric vector of Information Value contribution per bin.
- `count`: Integer vector of total observations per bin.
- `count_pos`: Integer vector of positive cases.
- `count_neg`: Integer vector of negative cases.
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `converged`: Logical indicating if the algorithm converged.
- `iterations`: Integer count of iterations performed.

### See Also

[ob\\_numerical\\_cm](#)

### Examples

```
# Example: Binning a small dataset where Fisher's Exact Test excels
set.seed(123)
feature <- rnorm(100)
target <- rbinom(100, 1, 0.2)

result <- ob_numerical_fetb(feature, target,
  min_bins = 2,
  max_bins = 4,
  max_n_prebins = 10
)

print(result$bin)
print(result$woe)
```

ob\_numerical\_ir

*Optimal Binning using Isotonic Regression (PAVA)***Description**

Performs supervised discretization of continuous numerical variables using Isotonic Regression (specifically the Pool Adjacent Violators Algorithm - PAVA). This method ensures a strictly monotonic relationship between bin indices and the empirical event rate, making it ideal for applications requiring shape constraints like credit scoring.

**Usage**

```
ob_numerical_ir(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  auto_monotonicity = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

**Arguments**

<code>feature</code>	A numeric vector representing the continuous predictor variable. Missing values (NA) are excluded from the binning process.
<code>target</code>	An integer vector of binary outcomes (0/1) corresponding to each observation in <code>feature</code> . Must have the same length as <code>feature</code> .
<code>min_bins</code>	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
<code>max_bins</code>	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
<code>bin_cutoff</code>	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins with frequency $< \text{bin\_cutoff}$ will be merged with neighbors. Value must be in (0, 1). Defaults to 0.05.
<code>max_n_prebins</code>	Integer. The number of initial quantiles to generate during the pre-binning phase. Defaults to 20.
<code>auto_monotonicity</code>	Logical. If TRUE, the algorithm automatically determines the optimal monotonicity direction (increasing or decreasing) based on the Pearson correlation between feature values and target. If FALSE, defaults to increasing monotonicity. Defaults to TRUE.
<code>convergence_threshold</code>	Numeric. Reserved for future use. Currently not actively used by the PAVA algorithm, which has guaranteed convergence. Defaults to 1e-6.

`max_iterations` Integer. Safety limit for iterative merging operations during pre-processing steps (e.g., rare bin merging). Defaults to 1000.

## Details

This function implements a **shape-constrained** binning approach using **Isotonic Regression**. Unlike heuristic merging strategies (ChiMerge, DP), this method finds the optimal monotonic fit in a single pass.

**Core Algorithm (PAVA):** The Pool Adjacent Violators Algorithm (Best & Chakravarti, 1990) is used to transform the empirical event rates of initial bins into a sequence that is either monotonically increasing or decreasing. It works by scanning the sequence and merging ("pooling") any adjacent pairs that violate the desired trend until a perfect fit is achieved. This guarantees an optimal solution in  $O(n)$  time.

### Process Flow:

1. **Pre-binning:** Creates initial bins using quantiles.
2. **Stabilization:** Merges bins below `bin_cutoff`.
3. **Trend Detection:** If `auto_monotonicity` = TRUE, calculates the correlation between feature midpoints and bin event rates to determine if the relationship should be increasing or decreasing.
4. **Shape Enforcement:** Applies PAVA to the sequence of bin event rates, producing a new set of rates that conform exactly to the monotonic constraint.
5. **Metric Calculation:** Derives WoE and IV from the adjusted rates.

### Advantages:

- **Global Optimality:** PAVA finds the best fit under the monotonicity constraint.
- **No Hyperparameters:** Unlike ChiMerge's p-value threshold, PAVA requires no significance level tuning for the core regression step.
- **Robustness:** Less sensitive to arbitrary thresholds compared to greedy merging.

## Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `iv`: Numeric vector of Information Value contribution per bin.
- `count`: Integer vector of total observations per bin.
- `count_pos`: Integer vector of positive cases.
- `count_neg`: Integer vector of negative cases.
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `total_iv`: The total Information Value of the binned variable.
- `monotone_increasing`: Logical indicating if the final WoE trend is increasing.
- `converged`: Logical indicating successful completion.

## References

Barlow, R. E., Bartholomew, D. J., Bremner, J. M., & Brunk, H. D. (1972). *Statistical inference under order restrictions*. John Wiley & Sons.

Best, M. J., & Chakravarti, N. (1990). Active set algorithms for isotonic regression; A unifying framework. *Mathematical Programming*, 47(1-3), 425-439.

## See Also

[ob\\_numerical\\_dp](#) for greedy dynamic programming approaches.

## Examples

```
# Example: Forcing a monotonic WoE trend
set.seed(123)
feature <- rnorm(500)
# Create a slightly noisy but generally increasing relationship
prob <- plogis(0.5 * feature + rnorm(500, 0, 0.3))
target <- rbinom(500, 1, prob)

result <- ob_numerical_ir(feature, target,
  min_bins = 4,
  max_bins = 6,
  auto_monotonicity = TRUE
)

print(result$bin)
print(round(result$woe, 3))
print(paste("Monotonic Increasing:", result$monotone_increasing))
```

---

<b>ob_numerical_jedi</b>	<i>Optimal Binning using Joint Entropy-Driven Interval Discretization (JEDI)</i>
--------------------------	--

---

## Description

Performs supervised discretization of continuous numerical variables using a holistic approach that balances entropy reduction (information gain) with statistical stability. The JEDI algorithm combines quantile-based initialization with an iterative optimization process that enforces monotonicity and minimizes Information Value (IV) loss.

## Usage

```
ob_numerical_jedi(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
```

```

  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) should be handled prior to binning.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins smaller than this threshold are merged. Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. The number of initial quantiles to generate during the initialization phase. Defaults to 20.
convergence_threshold	Numeric. The threshold for the change in total IV to determine convergence during the iterative optimization. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of iterations. Defaults to 1000.

## Details

The JEDI algorithm is designed to be a robust "all-rounder" for credit scoring and risk modeling. Its methodology proceeds in four distinct stages:

1. **Initialization (Quantile Pre-binning):** The feature space is divided into `max_n_prebins` segments containing approximately equal numbers of observations. This ensures the algorithm starts with a statistically balanced view of the data.
2. **Stabilization (Rare Bin Merging):** Adjacent bins with frequencies below `bin_cutoff` are merged. The merge direction is chosen to minimize the distortion of the event rate (similar to ChiMerge).
3. **Monotonicity Enforcement:** The algorithm heuristically determines the dominant trend (increasing or decreasing) of the Weight of Evidence (WoE) and iteratively merges adjacent bins that violate this trend. This step effectively reduces the conditional entropy of the binning sequence with respect to the target.
4. **IV Optimization:** If the number of bins exceeds `max_bins`, the algorithm merges the pair of adjacent bins that results in the smallest decrease in total Information Value. This greedy approach ensures that the final discretization retains the maximum possible predictive power given the constraints.

This joint approach (Entropy/IV + Stability constraints) makes JEDI particularly effective for datasets with noise or non-monotonic initial distributions that require smoothing.

**Value**

A list containing the binning results:

- **id**: Integer vector of bin identifiers.
- **bin**: Character vector of bin labels in interval notation.
- **woe**: Numeric vector of Weight of Evidence for each bin.
- **iv**: Numeric vector of Information Value contribution per bin.
- **count**: Integer vector of total observations per bin.
- **count\_pos**: Integer vector of positive cases.
- **count\_neg**: Integer vector of negative cases.
- **cutpoints**: Numeric vector of upper boundaries (excluding Inf).
- **converged**: Logical indicating if the algorithm converged.
- **iterations**: Integer count of iterations performed.

**See Also**

[ob\\_numerical\\_cm](#), [ob\\_numerical\\_ir](#)

**Examples**

```
# Example: Binning a variable with a complex relationship
set.seed(123)
feature <- rnorm(1000)
# Target probability has a quadratic component (non-monotonic)
# JEDI will try to force a monotonic approximation that maximizes IV
target <- rbinom(1000, 1, plogis(0.5 * feature + 0.1 * feature^2))

result <- ob_numerical_jedi(feature, target,
  min_bins = 3,
  max_bins = 6,
  max_n_prebins = 20
)
print(result$bin)
```

**Description**

Performs supervised discretization of continuous numerical variables for **multiclass** target variables (e.g., 0, 1, 2). It extends the Joint Entropy-Driven Interval (JEDI) discretization framework to calculate and optimize the Multinomial Weight of Evidence (M-WOE) for each class simultaneously.

## Usage

```
ob_numerical_jedi_mwoe(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) should be excluded prior to execution.
target	An integer vector of multiclass outcomes (0, 1, ..., K-1) corresponding to each observation in feature. Must have at least 2 distinct classes.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins smaller than this threshold are merged. Defaults to 0.05.
max_n_prebins	Integer. The number of initial quantiles to generate during the pre-binning phase. Defaults to 20.
convergence_threshold	Numeric. The threshold for the change in total Multinomial IV to determine convergence. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of iterations. Defaults to 1000.

## Details

**Multinomial Weight of Evidence (M-WOE):** For a target with  $K$  classes, the WoE for class  $k$  in bin  $i$  is defined using a "One-vs-Rest" approach:

$$WOE_{i,k} = \ln \left( \frac{P(X \in \text{bin}_i | Y = k)}{P(X \in \text{bin}_i | Y \neq k)} \right)$$

### Algorithm Workflow:

1. **Multiclass Initialization:** The algorithm starts with quantile-based bins and computes the initial event rates for all  $K$  classes.
2. **Joint Monotonicity:** The algorithm attempts to enforce monotonicity for *all* classes. If bin  $i$  violates the trend for Class 1 OR Class 2, it may be merged. This ensures the variable is predictive across the entire spectrum of outcomes.

3. **Global IV Optimization:** When reducing the number of bins to `max_bins`, the algorithm merges the pair of bins that minimizes the loss of the *Sum of IVs* across all classes:

$$Loss = \sum_{k=0}^{K-1} \Delta IV_k$$

This method is ideal for use cases like:

- predicting loan status (Current, Late, Default)
- customer churn levels (Active, Dormant, Churned)
- ordinal survey responses.

### Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: A numeric matrix where each column represents the WoE for a specific class (One-vs-Rest).
- `iv`: A numeric matrix where each column represents the IV contribution for a specific class.
- `count`: Integer vector of total observations per bin.
- `class_counts`: A matrix of observation counts per class per bin.
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `n_classes`: The number of distinct target classes found.

### See Also

[ob\\_numerical\\_jedi](#) for the binary version.

### Examples

```
# Example: Multiclass target (0, 1, 2)
set.seed(123)
feature <- rnorm(1000)
# Class 0: low feature, Class 1: medium, Class 2: high
target <- cut(feature + rnorm(1000, 0, 0.5),
               breaks = c(-Inf, -0.5, 0.5, Inf),
               labels = FALSE
) - 1

result <- ob_numerical_jedi_mwoe(feature, target,
                                    min_bins = 3,
                                    max_bins = 5
)

# Check WoE for Class 2 (High values)
print(result$woe[, 3]) # Column 3 corresponds to Class 2
```

---

ob_numerical_kmb	<i>Optimal Binning using K-means Inspired Initialization (KMB)</i>
------------------	--

---

## Description

Performs supervised discretization of continuous numerical variables using a K-means inspired binning strategy. Initial bin boundaries are determined by placing centroids uniformly across the feature range and defining cuts at midpoints. The algorithm then optimizes these bins using statistical constraints.

## Usage

```
ob_numerical_kmb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  enforce_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

<code>feature</code>	A numeric vector representing the continuous predictor variable. Missing values (NA) should be handled prior to binning.
<code>target</code>	An integer vector of binary outcomes (0/1) corresponding to each observation in <code>feature</code> . Must have the same length as <code>feature</code> .
<code>min_bins</code>	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
<code>max_bins</code>	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
<code>bin_cutoff</code>	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins smaller than this threshold are merged. Value must be in (0, 1). Defaults to 0.05.
<code>max_n_prebins</code>	Integer. The number of initial centroids/bins to generate during the initialization phase. Defaults to 20.
<code>enforce_monotonic</code>	Logical. If TRUE, the algorithm enforces a monotonic relationship in the Weight of Evidence (WoE) across bins. Defaults to TRUE.
<code>convergence_threshold</code>	Numeric. The threshold for determining convergence during the iterative optimization process. Defaults to 1e-6.
<code>max_iterations</code>	Integer. Safety limit for the maximum number of iterations. Defaults to 1000.

## Details

The KMB algorithm offers a unique initialization strategy compared to standard binning methods:

1. **Initialization (K-means Style):** Instead of using quantiles, `max_n_prebins` centroids are placed uniformly across the range  $[min(x), max(x)]$ . Bin boundaries are then defined as the midpoints between adjacent centroids. This can lead to more evenly distributed initial bin widths in terms of the feature's scale.
2. **Optimization:** The initialized bins undergo standard post-processing:
  - **Rare Bin Merging:** Bins below `bin_cutoff` are merged with their most similar neighbor (by event rate).
  - **Monotonicity:** If `enforce_monotonic = TRUE`, adjacent bins violating the dominant WoE trend are merged.
  - **Bin Count Adjustment:** If the number of bins exceeds `max_bins`, the algorithm greedily merges adjacent bins with the smallest absolute difference in Information Value.

This method can be advantageous when the underlying distribution of the feature is relatively uniform, as it avoids creating overly granular bins in dense regions from the start.

## Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `iv`: Numeric vector of Information Value contribution per bin.
- `count`: Integer vector of total observations per bin.
- `count_pos`: Integer vector of positive cases.
- `count_neg`: Integer vector of negative cases.
- `centroids`: Numeric vector of bin centroids (mean feature value per bin).
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `total_iv`: The total Information Value of the binned variable.
- `converged`: Logical indicating if the algorithm converged.

## See Also

[ob\\_numerical\\_ewb](#), [ob\\_numerical\\_cm](#)

## Examples

```
# Example: Comparing KMB with EWB on uniform data
set.seed(123)
feature <- runif(1000, 0, 100)
target <- rbinom(1000, 1, plogis(0.02 * feature))

result_kmb <- ob_numerical_kmb(feature, target, max_bins = 5)
```

```
print(result_kmb$bin)
print(paste("KMB Total IV:", round(result_kmb$total_iv, 4)))
```

## Description

Implements supervised discretization via Local Density Binning (LDB), a method that leverages kernel density estimation to identify natural transition regions in the feature space while optimizing the Weight of Evidence (WoE) monotonicity and Information Value (IV) for binary classification tasks.

## Usage

```
ob_numerical_ldb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  enforce_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	Numeric vector of feature values to be binned. Missing values (NA) and infinite values are automatically filtered out during preprocessing.
target	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as feature.
min_bins	Minimum number of bins to generate (default: 3). Must be at least 2.
max_bins	Maximum number of bins to generate (default: 5). Must be greater than or equal to min_bins.
bin_cutoff	Minimum fraction of total observations in each bin (default: 0.05). Bins with frequency below this threshold are merged with adjacent bins. Must be in the range [0, 1].
max_n_prebins	Maximum number of pre-bins before optimization (default: 20). Controls granularity of initial density-based discretization.
enforce_monotonic	Logical flag to enforce monotonicity in WoE values across bins (default: TRUE). When enabled, bins violating monotonicity are iteratively merged until global monotonicity is achieved.

```

convergence_threshold
    Convergence threshold for iterative optimization (default: 1e-6). Currently used
    for future extensions.

max_iterations Maximum number of iterations for merging operations (default: 1000). Prevents
    infinite loops in edge cases.

```

## Details

### Algorithm Overview

The Local Density Binning (LDB) algorithm operates in four sequential phases:

#### Phase 1: Density-Based Pre-binning

The algorithm employs kernel density estimation (KDE) with a Gaussian kernel to identify the local density structure of the feature:

$$\hat{f}(x) = \frac{1}{nh\sqrt{2\pi}} \sum_{i=1}^n \exp \left[ -\frac{(x - x_i)^2}{2h^2} \right]$$

where  $h$  is the bandwidth computed via Silverman's rule of thumb:

$$h = 0.9 \times \min(\hat{\sigma}, \text{IQR}/1.34) \times n^{-1/5}$$

Bin boundaries are placed at local minima of  $\hat{f}(x)$ , which correspond to natural transition regions where density is lowest (analogous to valleys in the density landscape). This strategy ensures bins capture homogeneous subpopulations.

#### Phase 2: Weight of Evidence Computation

For each bin  $i$ , the WoE quantifies the log-ratio of positive to negative class distributions, adjusted with Laplace smoothing ( $\alpha = 0.5$ ) to prevent division by zero:

$$\text{WoE}_i = \ln \left( \frac{\text{DistGood}_i}{\text{DistBad}_i} \right)$$

where:

$$\text{DistGood}_i = \frac{n_i^+ + \alpha}{n^+ + K\alpha}, \quad \text{DistBad}_i = \frac{n_i^- + \alpha}{n^- + K\alpha}$$

and  $K$  is the total number of bins. The Information Value for bin  $i$  is:

$$\text{IV}_i = (\text{DistGood}_i - \text{DistBad}_i) \times \text{WoE}_i$$

Total IV aggregates discriminatory power:  $\text{IV}_{\text{total}} = \sum_{i=1}^K \text{IV}_i$ .

#### Phase 3: Monotonicity Enforcement

When `enforce_monotonic` = TRUE, the algorithm ensures WoE values are monotonic with respect to bin order. The direction (increasing/decreasing) is determined via Pearson correlation between bin indices and WoE values. Bins violating monotonicity are iteratively merged using the `merge`

strategy described in Phase 4, continuing until global monotonicity is achieved or `min_bins` is reached.

This approach is rooted in isotonic regression principles (Robertson et al., 1988), ensuring the scorecard maintains a consistent logical relationship between feature values and credit risk.

#### Phase 4: Adaptive Bin Merging

Two merging criteria are applied sequentially:

1. **Frequency-based merging:** Bins with total count below `bin_cutoff`  $\times n$  are merged with the adjacent bin having the most similar event rate (minimizing heterogeneity). If event rates are equivalent, the merge that preserves higher IV is preferred.
2. **Cardinality reduction:** If the number of bins exceeds `max_bins`, the pair of adjacent bins minimizing IV loss when merged is identified via:

$$\Delta IV_{i,i+1} = IV_i + IV_{i+1} - IV_{\text{merged}}$$

This greedy optimization continues until  $K \leq \text{max\_bins}$ .

#### Theoretical Foundations

- **Kernel Density Estimation:** The bandwidth selection follows Silverman (1986, Chapter 3), balancing bias-variance tradeoff for univariate density estimation.
- **Weight of Evidence:** Siddiqi (2006) formalizes WoE/IV as measures of predictive strength in credit scoring, with IV thresholds: < 0.02 (unpredictive), 0.02-0.1 (weak), 0.1-0.3 (medium), 0.3-0.5 (strong), > 0.5 (suspect overfitting).
- **Supervised Discretization:** García et al. (2013) categorize LDB within "static" supervised methods that do not require iterative feedback from the model, unlike dynamic methods (e.g., ChiMerge).

#### Computational Complexity

- KDE computation:  $O(n^2)$  for naive implementation (each of  $n$  points evaluates  $n$  kernel terms).
- Binary search for bin assignment:  $O(n \log K)$  where  $K$  is the number of bins.
- Merge iterations:  $O(K^2 \times \text{max\_iterations})$  in worst case.

For large datasets ( $n > 10^5$ ), the KDE phase dominates runtime.

#### Value

A list containing:

- id** Integer vector of bin identifiers (1-based indexing).
- bin** Character vector of bin intervals in the format "(lower;upper]".
- woe** Numeric vector of Weight of Evidence values for each bin.
- iv** Numeric vector of Information Value contributions for each bin.
- count** Integer vector of total observations in each bin.
- count\_pos** Integer vector of positive class (`target = 1`) counts per bin.

**count\_neg** Integer vector of negative class (target = 0) counts per bin.  
**event\_rate** Numeric vector of event rates (proportion of positives) per bin.  
**cutpoints** Numeric vector of cutpoints defining bin boundaries (excluding -Inf and +Inf).  
**converged** Logical flag indicating whether the algorithm converged within **max\_iterations**.  
**iterations** Integer count of iterations performed during optimization.  
**total\_iv** Numeric scalar representing the total Information Value (sum of all bin IVs).  
**monotonicity** Character string indicating monotonicity status: "increasing", "decreasing", or "none".

### Author(s)

Lopes, J. E. (implemented algorithm)

### References

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- Dougherty, J., Kohavi, R., & Sahami, M. (1995). "Supervised and Unsupervised Discretization of Continuous Features". *Proceedings of the 12th International Conference on Machine Learning*, pp. 194-202.
- Robertson, T., Wright, F. T., & Dykstra, R. L. (1988). *Order Restricted Statistical Inference*. Wiley.
- García, S., Luengo, J., Sáez, J. A., López, V., & Herrera, F. (2013). "A Survey of Discretization Techniques: Taxonomy and Empirical Analysis in Supervised Learning". *IEEE Transactions on Knowledge and Data Engineering*, 25(4), 734-750.

### See Also

[ob\\_numerical\\_mdlp](#) for Minimum Description Length Principle binning, [ob\\_numerical\\_mob](#) for monotonic binning with similar constraints.

### Examples

```

# Simulate credit scoring data
set.seed(42)
n <- 10000
feature <- c(
  rnorm(3000, mean = 600, sd = 50), # Low-risk segment
  rnorm(4000, mean = 700, sd = 40), # Medium-risk segment
  rnorm(3000, mean = 750, sd = 30) # High-risk segment
)
target <- c(
  rbinom(3000, 1, 0.15), # 15% default rate
  rbinom(4000, 1, 0.08), # 8% default rate
  rbinom(3000, 1, 0.03) # 3% default rate
)
  
```

```

)
# Apply LDB with monotonicity enforcement
result <- ob_numerical_ldb(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  enforce_monotonic = TRUE
)

# Inspect binning quality
print(result$total_iv) # Should be > 0.1 for predictive features
print(result$monotonicity) # Should indicate direction

# Visualize WoE pattern
plot(result$woe,
  type = "b", xlab = "Bin", ylab = "WoE",
  main = "Monotonic WoE Trend"
)

# Generate scorecard transformation
bin_mapping <- data.frame(
  bin = result$bin,
  woe = result$woe,
  iv = result$iv
)
print(bin_mapping)

```

---

ob\_numerical\_lpdb

*Optimal Binning using Local Polynomial Density Binning (LPDB)*

---

## Description

Performs supervised discretization of continuous numerical variables using a novel approach that combines non-parametric density estimation with information-theoretic optimization. The algorithm first identifies natural clusters and boundaries in the feature distribution using local polynomial density estimation, then refines the bins to maximize predictive power.

## Usage

```

ob_numerical_lpdb(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,

```

```

  bin_cutoff = 0.05,
  max_n_prebins = 20,
  polynomial_degree = 3,
  enforce_monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)

```

## Arguments

feature	A numeric vector representing the continuous predictor variable. Missing values (NA) should be handled prior to binning.
target	An integer vector of binary outcomes (0/1) corresponding to each observation in feature. Must have the same length as feature.
min_bins	Integer. The minimum number of bins to produce. Must be $\geq 2$ . Defaults to 3.
max_bins	Integer. The maximum number of bins to produce. Must be $\geq \text{min\_bins}$ . Defaults to 5.
bin_cutoff	Numeric. The minimum fraction of total observations required for a bin to be considered valid. Bins smaller than this threshold are merged. Value must be in (0, 1). Defaults to 0.05.
max_n_prebins	Integer. The maximum number of initial candidate cut points to generate during the density estimation phase. Defaults to 20.
polynomial_degree	Integer. The degree of the local polynomial used for density estimation (note: currently approximated via KDE). Defaults to 3.
enforce_monotonic	Logical. If TRUE, the algorithm forces the Weight of Evidence (WoE) trend to be strictly monotonic. Defaults to TRUE.
convergence_threshold	Numeric. The threshold for determining convergence during the iterative merging process. Defaults to 1e-6.
max_iterations	Integer. Safety limit for the maximum number of merging iterations. Defaults to 1000.

## Details

The **Local Polynomial Density Binning (LPDB)** algorithm is a two-stage process:

### 1. Density-Based Initialization:

- Estimates the probability density function  $f(x)$  of the feature using Kernel Density Estimation (KDE), which approximates local polynomial regression.
- Identifies *critical points* on the density curve, such as local minima and inflection points. These points often correspond to natural boundaries between clusters or modes in the data.
- Uses these critical points as initial candidate cut points to form pre-bins.

### 2. Supervised Refinement:

- Calculates WoE and IV for each pre-bin.
- Enforces monotonicity by merging bins that violate the trend (determined by the correlation between bin centroids and WoE values).
- Merges bins with frequencies below `bin_cutoff`.
- Iteratively merges bins to meet the `max_bins` constraint, choosing merges that minimize the loss of total Information Value.

This method is particularly powerful for complex, multi-modal distributions where standard quantile or equal-width binning might obscure important structural breaks.

### Value

A list containing the binning results:

- `id`: Integer vector of bin identifiers.
- `bin`: Character vector of bin labels in interval notation.
- `woe`: Numeric vector of Weight of Evidence for each bin.
- `iv`: Numeric vector of Information Value contribution per bin.
- `count`: Integer vector of total observations per bin.
- `count_pos`: Integer vector of positive cases.
- `count_neg`: Integer vector of negative cases.
- `event_rate`: Numeric vector of the target event rate in each bin.
- `centroids`: Numeric vector of the geometric centroids of the final bins.
- `cutpoints`: Numeric vector of upper boundaries (excluding Inf).
- `total_iv`: The total Information Value of the binned variable.
- `monotonicity`: Character string indicating the final WoE trend ("increasing", "decreasing", or "none").

### See Also

[ob\\_numerical\\_kmb](#), [ob\\_numerical\\_jedi](#)

### Examples

```
# Example: Binning a tri-modal distribution
set.seed(123)
# Feature with three distinct clusters
feature <- c(rnorm(300, mean = -3), rnorm(400, mean = 0), rnorm(300, mean = 3))
# Target depends on these clusters
target <- rbinom(1000, 1, plogis(feature))

result <- ob_numerical_lpdb(feature, target,
  min_bins = 3,
  max_bins = 5
)

print(result$bin) # Should ideally find cuts near -1.5 and 1.5
print(result$monotonicity)
```

ob\_numerical\_mblp

*Optimal Binning for Numerical Features Using Monotonic Binning via Linear Programming*

## Description

Implements a greedy optimization algorithm for supervised discretization of numerical features with **guaranteed monotonicity** in Weight of Evidence (WoE). Despite the "Linear Programming" designation, this method employs an iterative heuristic based on quantile pre-binning, Information Value (IV) optimization, and monotonicity enforcement through adaptive bin merging.

**Important Note:** This algorithm does not use formal Linear Programming solvers (e.g., simplex method). The name reflects the conceptual formulation of binning as a constrained optimization problem, but the implementation uses a deterministic greedy heuristic for computational efficiency.

## Usage

```
ob_numerical_mblp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  force_monotonic_direction = 0,
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

feature	Numeric vector of feature values to be binned. Missing values (NA) and infinite values are automatically removed during preprocessing.
target	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as feature.
min_bins	Minimum number of bins to generate (default: 3). Must be at least 2.
max_bins	Maximum number of bins to generate (default: 5). Must be greater than or equal to min_bins.
bin_cutoff	Minimum fraction of total observations in each bin (default: 0.05). Bins with frequency below this threshold are merged with adjacent bins. Must be in the range (0, 1).
max_n_prebins	Maximum number of pre-bins before optimization (default: 20). Controls granularity of initial quantile-based discretization.
force_monotonic_direction	Integer flag to force a specific monotonicity direction (default: 0). Valid values:

- 0: Automatically determine direction via correlation between bin indices and WoE values.
- 1: Force increasing monotonicity (WoE increases with feature value).
- -1: Force decreasing monotonicity (WoE decreases with feature value).

convergence\_threshold

Convergence threshold for iterative optimization (default: 1e-6). Iteration stops when the absolute change in total IV between consecutive iterations falls below this value.

max\_iterations Maximum number of iterations for the optimization loop (default: 1000). Prevents infinite loops in pathological cases.

## Details

### Algorithm Overview

The Monotonic Binning via Linear Programming (MBLP) algorithm operates in four sequential phases designed to balance predictive power (IV maximization) and interpretability (monotonic WoE):

#### Phase 1: Quantile-Based Pre-binning

Initial bin boundaries are determined using empirical quantiles of the feature distribution. For  $k$  pre-bins, cutpoints are computed as:

$$q_i = x_{(\lceil p_i \times (N-1) \rceil)}, \quad p_i = \frac{i}{k}, \quad i = 1, 2, \dots, k-1$$

where  $x_{(j)}$  denotes the  $j$ -th order statistic. This approach ensures equal-frequency bins under the assumption of continuous data, though ties may cause deviations in practice. The first and last boundaries are set to  $-\infty$  and  $+\infty$ , respectively.

#### Phase 2: Frequency-Based Bin Merging

Bins with total count below  $\text{bin\_cutoff} \times N$  are iteratively merged with adjacent bins to ensure statistical reliability. The merge strategy selects the neighbor with the smallest count (greedy heuristic), continuing until all bins meet the frequency threshold or  $\text{min\_bins}$  is reached.

#### Phase 3: Monotonicity Direction Determination

If  $\text{force\_monotonic\_direction} = 0$ , the algorithm computes the Pearson correlation between bin indices and WoE values:

$$\rho = \frac{\sum_{i=1}^k (i - \bar{i})(\text{WoE}_i - \bar{\text{WoE}})}{\sqrt{\sum_{i=1}^k (i - \bar{i})^2 \sum_{i=1}^k (\text{WoE}_i - \bar{\text{WoE}})^2}}$$

The monotonicity direction is set as:

$$\text{direction} = \begin{cases} 1 & \text{if } \rho \geq 0 \text{ (increasing)} \\ -1 & \text{if } \rho < 0 \text{ (decreasing)} \end{cases}$$

If  $\text{force\_monotonic\_direction}$  is explicitly set to 1 or -1, that value overrides the correlation-based determination.

#### Phase 4: Iterative Optimization Loop

The core optimization alternates between two enforcement steps until convergence:

1. **Cardinality Constraint:** If the number of bins  $k$  exceeds `max_bins`, the algorithm identifies the pair of adjacent bins  $(i, i + 1)$  that minimizes the IV loss when merged:

$$\Delta IV_{i,i+1} = IV_i + IV_{i+1} - IV_{\text{merged}}$$

where  $IV_{\text{merged}}$  is recalculated using combined counts. The merge is performed only if it preserves monotonicity (checked via WoE comparison with neighboring bins).

2. **Monotonicity Enforcement:** For each pair of consecutive bins, violations are detected as:

- **Increasing:**  $\text{WoE}_i < \text{WoE}_{i-1} - \epsilon$
- **Decreasing:**  $\text{WoE}_i > \text{WoE}_{i-1} + \epsilon$

where  $\epsilon = 10^{-10}$  (numerical tolerance). Violating bins are immediately merged.

3. **Convergence Test:** After each iteration, the total IV is compared to the previous iteration. If  $|IV^{(t)} - IV^{(t-1)}| < \text{convergence\_threshold}$  or monotonicity is achieved, the loop terminates.

#### Weight of Evidence Computation

WoE for bin  $i$  uses Laplace smoothing ( $\alpha = 0.5$ ) to handle zero counts:

$$\text{WoE}_i = \ln \left( \frac{\text{DistGood}_i}{\text{DistBad}_i} \right)$$

where:

$$\text{DistGood}_i = \frac{n_i^+ + \alpha}{n^+ + k\alpha}, \quad \text{DistBad}_i = \frac{n_i^- + \alpha}{n^- + k\alpha}$$

and  $k$  is the current number of bins. The Information Value contribution is:

$$IV_i = (\text{DistGood}_i - \text{DistBad}_i) \times \text{WoE}_i$$

#### Theoretical Foundations

- **Monotonicity Requirement:** Zeng (2014) proves that monotonic WoE is a necessary condition for stable scorecards under data drift. Non-monotonic patterns often indicate overfitting to noise.
- **Greedy Optimization:** Unlike global optimizers (MILP), greedy heuristics provide no optimality guarantees but achieve  $O(k^2)$  complexity per iteration versus exponential for exact methods.
- **Quantile Binning:** Ensures initial bins have approximately equal sample sizes, reducing variance in WoE estimates (especially critical for minority classes).

#### Comparison with True Linear Programming

Formal LP formulations for binning (Belotti et al., 2016) express the problem as:

$$\max_{\mathbf{z}, \mathbf{b}} \sum_{i=1}^k IV_i(\mathbf{b})$$

subject to:

$$\text{WoE}_i \leq \text{WoE}_{i+1} \quad \forall i \quad (\text{monotonicity})$$

$$\sum_{j=1}^N z_{ij} = 1 \quad \forall j \quad (\text{assignment})$$

$$z_{ij} \in \{0, 1\}, \quad b_i \in \mathbb{R}$$

where  $z_{ij}$  indicates if observation  $j$  is in bin  $i$ , and  $b_i$  are bin boundaries. Such formulations require MILP solvers (CPLEX, Gurobi) and scale poorly beyond  $N > 10^4$ . MBLP sacrifices global optimality for **scalability** and **determinism**.

### Computational Complexity

- Initial sorting:  $O(N \log N)$
- Quantile computation:  $O(k)$
- Per-iteration operations:  $O(k^2)$  (pairwise comparisons for merging)
- Total:  $O(N \log N + k^2 \times \text{max\_iterations})$

For typical credit scoring datasets ( $N \sim 10^5$ ,  $k \sim 5$ ), runtime is dominated by sorting. Pathological cases (highly non-monotonic data) may require many iterations to enforce monotonicity.

### Value

A list containing:

- id** Integer vector of bin identifiers (1-based indexing).
- bin** Character vector of bin intervals in the format "(lower;upper]".
- woe** Numeric vector of Weight of Evidence values for each bin.
- iv** Numeric vector of Information Value contributions for each bin.
- count** Integer vector of total observations in each bin.
- count\_pos** Integer vector of positive class (target = 1) counts per bin.
- count\_neg** Integer vector of negative class (target = 0) counts per bin.
- event\_rate** Numeric vector of event rates (proportion of positives) per bin.
- cutpoints** Numeric vector of cutpoints defining bin boundaries (excluding -Inf and +Inf).
- converged** Logical flag indicating whether the algorithm converged within `max_iterations`.
- iterations** Integer count of iterations performed during optimization.
- total\_iv** Numeric scalar representing the total Information Value (sum of all bin IVs).
- monotonicity** Character string indicating monotonicity status: "increasing", "decreasing", or "none".

### Author(s)

Lopes, J. E. (implemented algorithm based on Mironchyk & Tchistiakov, 2017)

## References

- Zeng, G. (2014). "A Necessary Condition for a Good Binning Algorithm in Credit Scoring". *Applied Mathematical Sciences*, 8(65), 3229-3242.
- Mironchyk, P., & Tchistiakov, V. (2017). "Monotone optimal binning algorithm for credit risk modeling". *Frontiers in Applied Mathematics and Statistics*, 3, 2.
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## See Also

[ob\\_numerical\\_ldb](#) for density-based binning, [ob\\_numerical\\_mdlp](#) for entropy-based discretization with MDLP criterion.

## Examples

```
# Simulate non-monotonic credit scoring data
set.seed(123)
n <- 8000
feature <- c(
  rnorm(2000, mean = 550, sd = 60), # High-risk segment (low scores)
  rnorm(3000, mean = 680, sd = 50), # Medium-risk segment
  rnorm(2000, mean = 720, sd = 40), # Low-risk segment
  rnorm(1000, mean = 620, sd = 55) # Mixed segment (creates non-monotonicity)
)
target <- c(
  rbinom(2000, 1, 0.25), # 25% default rate
  rbinom(3000, 1, 0.10), # 10% default rate
  rbinom(2000, 1, 0.03), # 3% default rate
  rbinom(1000, 1, 0.15) # 15% default rate (violates monotonicity)
)

# Apply MBLP with automatic monotonicity detection
result_auto <- ob_numerical_mblp(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  force_monotonic_direction = 0 # Auto-detect
)
```

```

print(result_auto$monotonicity) # Check detected direction
print(result_auto$total_iv) # Should be > 0.1 for predictive features

# Force decreasing monotonicity (higher score = lower WoE = lower risk)
result_forced <- ob_numerical_mdlp(
  feature = feature,
  target = target,
  min_bins = 4,
  max_bins = 6,
  force_monotonic_direction = -1 # Force decreasing
)

# Verify monotonicity enforcement
stopifnot(all(diff(result_forced$woe) <= 1e-9)) # Should be non-increasing

# Compare convergence
cat(sprintf(
  "Auto mode: %d iterations, IV = %.4f\n",
  result_auto$iterations, result_auto$total_iv
))
cat(sprintf(
  "Forced mode: %d iterations, IV = %.4f\n",
  result_forced$iterations, result_forced$total_iv
))

# Visualize binning quality
oldpar <- par(mfrow = c(1, 2))
plot(result_auto$woe,
  type = "b", col = "blue", pch = 19,
  xlab = "Bin", ylab = "WoE", main = "Auto-Detected Monotonicity"
)
plot(result_forced$woe,
  type = "b", col = "red", pch = 19,
  xlab = "Bin", ylab = "WoE", main = "Forced Decreasing"
)
par(oldpar)

```

## Description

Implements the Minimum Description Length Principle (MDLP) for supervised discretization of numerical features. MDLP balances model complexity (number of bins) and data fit (information gain) through a rigorous information-theoretic framework, automatically determining the optimal number of bins without arbitrary thresholds.

Unlike heuristic methods, MDLP provides a **theoretically grounded stopping criterion** based on the trade-off between encoding the binning structure and encoding the data given that structure. This makes it particularly robust against overfitting in noisy datasets.

## Usage

```
ob_numerical_mdlp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  laplace_smoothing = 0.5
)
```

## Arguments

<code>feature</code>	Numeric vector of feature values to be binned. Missing values (NA) are automatically removed during preprocessing. Infinite values trigger a warning but are handled internally.
<code>target</code>	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as <code>feature</code> .
<code>min_bins</code>	Minimum number of bins to generate (default: 3). Must be at least 1. If the number of unique feature values is less than <code>min_bins</code> , the algorithm adjusts automatically.
<code>max_bins</code>	Maximum number of bins to generate (default: 5). Must be greater than or equal to <code>min_bins</code> . Acts as a hard constraint after MDLP optimization.
<code>bin_cutoff</code>	Minimum fraction of total observations required in each bin (default: 0.05). Bins with frequency below this threshold are merged with adjacent bins to ensure statistical reliability. Must be in the range (0, 1).
<code>max_n_prebins</code>	Maximum number of pre-bins before MDLP optimization (default: 20). Higher values allow finer granularity but increase computational cost. Must be at least 2.
<code>convergence_threshold</code>	Convergence threshold for iterative optimization (default: 1e-6). Currently used internally for future extensions; MDLP convergence is primarily determined by the MDL cost function.
<code>max_iterations</code>	Maximum number of iterations for bin merging operations (default: 1000). Prevents infinite loops in pathological cases. A warning is issued if this limit is reached.
<code>laplace_smoothing</code>	Laplace smoothing parameter for WoE calculation (default: 0.5). Prevents division by zero and stabilizes WoE estimates in bins with zero counts for one class. Must be non-negative. Higher values increase regularization but may dilute signal in small bins.

## Details

### Algorithm Overview

The MDLP algorithm executes in five sequential phases:

#### Phase 1: Data Preparation and Validation

Input data is validated for:

- Binary target (only 0 and 1 values)
- Parameter consistency (`min_bins`  $\leq$  `max_bins`, valid ranges)
- Missing value detection (NaN/Inf are filtered out with a warning)

Feature-target pairs are sorted by feature value in ascending order, enabling efficient bin assignment via linear scan.

#### Phase 2: Equal-Frequency Pre-binning

Initial bins are created by dividing the sorted data into approximately equal-sized groups:

$$n_{\text{records/bin}} = \max \left( 1, \left\lfloor \frac{N}{\text{max\_n\_prebins}} \right\rfloor \right)$$

This ensures each pre-bin has sufficient observations for stable entropy estimation. Bin boundaries are set to feature values at split points, with first and last boundaries at  $-\infty$  and  $+\infty$ .

For each bin  $i$ , Shannon entropy is computed:

$$H(S_i) = -p_i \log_2(p_i) - q_i \log_2(q_i)$$

where  $p_i = n_i^+ / n_i$  (proportion of positives) and  $q_i = 1 - p_i$ . Pure bins ( $p_i = 0$  or  $p_i = 1$ ) have  $H(S_i) = 0$ .

**Performance Note:** Entropy calculation uses a precomputed lookup table for bin counts 0-100, achieving 30-50% speedup compared to runtime computation.

#### Phase 3: MDL-Based Greedy Merging

The core optimization minimizes the Minimum Description Length, defined as:

$$\text{MDL}(k) = L_{\text{model}}(k) + L_{\text{data}}(k)$$

where:

- **Model Cost:**  $L_{\text{model}}(k) = \log_2(k - 1)$   
Encodes the number of bins. Increases logarithmically with bin count, penalizing complex models.
- **Data Cost:**  $L_{\text{data}}(k) = N \cdot H(S_{\text{total}}) - \sum_{i=1}^k n_i \cdot H(S_i)$   
Measures unexplained uncertainty after binning. Lower values indicate better class separation.

The algorithm iteratively evaluates all  $k - 1$  adjacent bin pairs, computing  $\text{MDL}(k - 1)$  for each potential merge. The pair minimizing MDL cost is merged, continuing until:

1.  $k = \text{min\_bins}$ , or

2. No merge reduces MDL cost (local optimum), or
3. `max_iterations` is reached

**Theoretical Guarantee** (Fayyad & Irani, 1993): The MDL criterion provides a \*\*consistent estimator\*\* of the true discretization complexity under mild regularity conditions, unlike ad-hoc stopping rules.

#### Phase 4: Rare Bin Handling

Bins with frequency  $n_i/N < \text{bin\_cutoff}$  are merged with adjacent bins. The merge direction (left or right) is chosen by minimizing post-merge entropy:

$$\text{direction} = \arg \min_{d \in \{\text{left, right}\}} H(S_i \cup S_{i+d})$$

This preserves class homogeneity while ensuring statistical reliability.

#### Phase 5: Monotonicity Enforcement (Optional)

If WoE values violate monotonicity ( $\text{WoE}_i < \text{WoE}_{i-1}$ ), bins are iteratively merged until:

$$\text{WoE}_1 \leq \text{WoE}_2 \leq \dots \leq \text{WoE}_k$$

Merge decisions prioritize preserving Information Value:

$$\Delta \text{IV} = \text{IV}_i + \text{IV}_{i+1} - \text{IV}_{\text{merged}}$$

Merges proceed only if  $\text{IV}_{\text{merged}} \geq 0.5 \times (\text{IV}_i + \text{IV}_{i+1})$ .

#### Weight of Evidence Computation

WoE for bin  $i$  includes Laplace smoothing to handle zero counts:

$$\text{WoE}_i = \ln \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} \middle/ \frac{n_i^- + \alpha}{n^- + k\alpha} \right)$$

where  $\alpha = \text{laplace\_smoothing}$  and  $k$  is the number of bins.

#### Edge cases:

- If  $n_i^+ + \alpha = n_i^- + \alpha = 0$ :  $\text{WoE}_i = 0$
- If  $n_i^+ + \alpha = 0$ :  $\text{WoE}_i = -20$  (capped)
- If  $n_i^- + \alpha = 0$ :  $\text{WoE}_i = +20$  (capped)

Information Value is computed as:

$$\text{IV}_i = \left( \frac{n_i^+}{n^+} - \frac{n_i^-}{n^-} \right) \times \text{WoE}_i$$

#### Comparison with Other Methods

Method	Stopping Criterion	Optimality
MDLP	Information-theoretic (MDL cost)	Local optimum with theoretical guarantees

LDB	Heuristic (density minima)	No formal optimality
MBLP	Heuristic (IV loss threshold)	Greedy approximation
ChiMerge	Statistical ( $\chi^2$ test)	Dependent on significance level

## Computational Complexity

- Sorting:  $O(N \log N)$
- Pre-binning:  $O(N)$
- MDL optimization:  $O(k^3 \times I)$  where  $I$  is the number of merge iterations (typically  $I \approx k$ )
- Total:  $O(N \log N + k^3 \times I)$

For typical credit scoring datasets ( $N \sim 10^5$ ,  $k \sim 5$ ), runtime is dominated by sorting.

## Value

A list containing:

- id** Integer vector of bin identifiers (1-based indexing).
- bin** Character vector of bin intervals in the format "[lower;upper)". The first bin starts with -Inf and the last bin ends with +Inf.
- woe** Numeric vector of Weight of Evidence values for each bin, computed with Laplace smoothing.
- iv** Numeric vector of Information Value contributions for each bin.
- count** Integer vector of total observations in each bin.
- count\_pos** Integer vector of positive class (target = 1) counts per bin.
- count\_neg** Integer vector of negative class (target = 0) counts per bin.
- cutpoints** Numeric vector of cutpoints defining bin boundaries (excluding -Inf and +Inf). These are the upper bounds of bins 1 to k-1.
- total\_iv** Numeric scalar representing the total Information Value (sum of all bin IVs).
- converged** Logical flag indicating whether the algorithm converged. Set to FALSE if `max_iterations` was reached during any merging phase.
- iterations** Integer count of iterations performed across all optimization phases (MDL merging, rare bin merging, monotonicity enforcement).

## Author(s)

Lopes, J. E. (algorithm implementation based on Fayyad & Irani, 1993)

## References

- Fayyad, U. M., & Irani, K. B. (1993). "Multi-Interval Discretization of Continuous-Valued Attributes for Classification Learning". *Proceedings of the 13th International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 1022-1027.
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- Witten, I. H., Frank, E., & Hall, M. A. (2011). *Data Mining: Practical Machine Learning Tools and Techniques* (3rd ed.). Morgan Kaufmann.
- Cerqueira, V., & Torgo, L. (2019). "Automatic Feature Engineering for Predictive Modeling of Multivariate Time Series". arXiv:1910.01344.

## See Also

[ob\\_numerical\\_ldb](#) for density-based binning, [ob\\_numerical\\_mblp](#) for monotonicity-constrained binning.

## Examples

```

# Simulate overdispersed credit scoring data with noise
set.seed(2024)
n <- 10000

# Create feature with multiple regimes and noise
feature <- c(
  rnorm(3000, mean = 580, sd = 70), # High-risk cluster
  rnorm(4000, mean = 680, sd = 50), # Medium-risk cluster
  rnorm(2000, mean = 740, sd = 40), # Low-risk cluster
  runif(1000, min = 500, max = 800) # Noise (uniform distribution)
)

target <- c(
  rbinom(3000, 1, 0.30), # 30% default rate
  rbinom(4000, 1, 0.12), # 12% default rate
  rbinom(2000, 1, 0.04), # 4% default rate
  rbinom(1000, 1, 0.15) # Noisy segment
)

# Apply MDLP with default parameters
result <- ob_numerical_mdlp(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20
)

# Inspect results
print(result$bin)
print(data.frame(
  Bin = result$bin,
  WoE = round(result$woe, 4),
  N = result$N,
  Min = result$Min,
  Max = result$Max
))

```

```

IV = round(result$iv, 4),
Count = result$count
))

cat(sprintf("\nTotal IV: %.4f\n", result$total_iv))
cat(sprintf("Converged: %s\n", result$converged))
cat(sprintf("Iterations: %d\n", result$iterations))

# Verify monotonicity
is_monotonic <- all(diff(result$woe) >= -1e-10)
cat(sprintf("WoE Monotonic: %s\n", is_monotonic))

# Compare with different Laplace smoothing
result_nosmooth <- ob_numerical_mdlp(
  feature = feature,
  target = target,
  laplace_smoothing = 0.0 # No smoothing (risky for rare bins)
)

result_highsmooth <- ob_numerical_mdlp(
  feature = feature,
  target = target,
  laplace_smoothing = 2.0 # Higher regularization
)

# Compare WoE stability
data.frame(
  Bin = seq_along(result$woe),
  WoE_default = result$woe,
  WoE_no_smooth = result_nosmooth$woe,
  WoE_high_smooth = result_highsmooth$woe
)

# Visualize binning structure
oldpar <- par(mfrow = c(1, 2))

# WoE plot
plot(result$woe,
  type = "b", col = "blue", pch = 19,
  xlab = "Bin", ylab = "WoE",
  main = "Weight of Evidence by Bin"
)
grid()

# IV contribution plot
barplot(result$iv,
  names.arg = seq_along(result$iv),
  col = "steelblue", border = "white",
  xlab = "Bin", ylab = "IV Contribution",
  main = sprintf("Total IV = %.4f", result$total_iv)
)
grid()
par(oldpar)

```

---

ob_numerical_mob	<i>Optimal Binning for Numerical Features using Monotonic Optimal Binning</i>
------------------	---

---

## Description

Implements Monotonic Optimal Binning (MOB), a supervised discretization algorithm that enforces strict monotonicity in Weight of Evidence (WoE) values. MOB is designed for credit scoring and risk modeling applications where monotonicity is a **regulatory requirement** or essential for model interpretability and stakeholder acceptance.

Unlike heuristic methods that treat monotonicity as a post-processing step, MOB integrates monotonicity constraints into the core optimization loop, ensuring that the final binning satisfies:  $\text{WoE}_1 \leq \text{WoE}_2 \leq \dots \leq \text{WoE}_k$  (or the reverse for decreasing patterns).

## Usage

```
ob_numerical_mob(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  laplace_smoothing = 0.5
)
```

## Arguments

feature	Numeric vector of feature values to be binned. Missing values (NA) are automatically removed during preprocessing. Infinite values trigger a warning but are handled internally.
target	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as feature.
min_bins	Minimum number of bins to generate (default: 3). Must be at least 2. Acts as a hard constraint during monotonicity enforcement; the algorithm will not merge below this threshold even if violations persist.
max_bins	Maximum number of bins to generate (default: 5). Must be greater than or equal to min_bins. The algorithm reduces bins via greedy merging if the initial count exceeds this limit.

bin_cutoff	Minimum fraction of total observations required in each bin (default: 0.05). Bins with frequency below this threshold are merged with adjacent bins. Must be in the range (0, 1).
max_n_prebins	Maximum number of pre-bins before optimization (default: 20). Controls granularity of initial equal-frequency discretization. Must be at least equal to <code>min_bins</code> .
convergence_threshold	Convergence threshold for iterative optimization (default: 1e-6). Reserved for future extensions; current implementation uses <code>max_iterations</code> as the primary stopping criterion.
max_iterations	Maximum number of iterations for bin merging and monotonicity enforcement (default: 1000). Prevents infinite loops in pathological cases. A warning is issued if this limit is reached without achieving convergence.
laplace_smoothing	Laplace smoothing parameter for WoE calculation (default: 0.5). Prevents division by zero and stabilizes WoE estimates in bins with zero counts for one class. Must be non-negative. Standard values: 0.5 (Laplace), 1.0 (Jeffreys prior).

## Details

### Algorithm Overview

The MOB algorithm executes in five sequential phases with strict monotonicity enforcement integrated throughout:

#### Phase 1: Equal-Frequency Pre-binning

Initial bins are created by dividing sorted data into approximately equal-sized groups:

$$n_{\text{bin}} = \left\lfloor \frac{N}{\min(\text{max\_n\_prebins}, n_{\text{unique}})} \right\rfloor$$

Bin boundaries are set to feature values at split points, ensuring no gaps between consecutive bins. First and last boundaries are set to  $-\infty$  and  $+\infty$ .

This approach balances statistical stability (sufficient observations per bin) with granularity (ability to detect local patterns).

#### Phase 2: Rare Bin Merging

Bins with total count below  $\text{bin\_cutoff} \times N$  are iteratively merged. The merge direction (left or right) is chosen to minimize Information Value loss:

$$\text{direction} = \arg \min_{d \in \{\text{left, right}\}} (\text{IV}_{\text{before}} - \text{IV}_{\text{after merge}})$$

where:

$$\text{IV}_{\text{before}} = \text{IV}_i + \text{IV}_{i+d}$$

$$\text{IV}_{\text{after}} = (\text{DistGood}_{\text{merged}} - \text{DistBad}_{\text{merged}}) \times \text{WoE}_{\text{merged}}$$

Merging continues until all bins meet the frequency threshold or `min_bins` is reached.

#### Phase 3: Initial WoE/IV Calculation

Weight of Evidence for each bin  $i$  is computed with Laplace smoothing:

$$\text{WoE}_i = \ln \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} \middle/ \frac{n_i^- + \alpha}{n^- + k\alpha} \right)$$

where  $\alpha$  = laplace\_smoothing and  $k$  is the current number of bins. Information Value is:

$$\text{IV}_i = \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} - \frac{n_i^- + \alpha}{n^- + k\alpha} \right) \times \text{WoE}_i$$

#### Edge case handling:

- If both distributions approach zero:  $\text{WoE}_i = 0$
- If only positive distribution is zero:  $\text{WoE}_i = -20$  (capped)
- If only negative distribution is zero:  $\text{WoE}_i = +20$  (capped)

#### Phase 4: Monotonicity Enforcement

The algorithm first determines the desired monotonicity direction by examining the relationship between the first two bins:

$$\text{should\_increase} = \begin{cases} \text{TRUE} & \text{if } \text{WoE}_1 \geq \text{WoE}_0 \\ \text{FALSE} & \text{otherwise} \end{cases}$$

For each bin  $i$  from 1 to  $k - 1$ , violations are detected as:

$$\text{violation} = \begin{cases} \text{WoE}_i < \text{WoE}_{i-1} & \text{if } \text{should\_increase} \\ \text{WoE}_i > \text{WoE}_{i-1} & \text{if } \neg \text{should\_increase} \end{cases}$$

When a violation is found at index  $i$ , the algorithm attempts two merge strategies:

1. **Merge with previous bin:** Combine bins  $i - 1$  and  $i$ , then verify the merged bin's WoE is compatible with neighbors:

$$\text{WoE}_{i-2} \leq \text{WoE}_{\text{merged}} \leq \text{WoE}_{i+1} \quad (\text{if } \text{should\_increase})$$

2. **Merge with next bin:** If strategy 1 fails, merge bins  $i$  and  $i + 1$ .

Merging continues iteratively until either:

- All WoE values satisfy monotonicity constraints
- The number of bins reaches `min_bins`
- `max_iterations` is exceeded (triggers warning)

After each merge, WoE and IV are recalculated for all bins to reflect updated distributions.

#### Phase 5: Bin Count Reduction

If the number of bins exceeds `max_bins` after monotonicity enforcement, additional merges are performed. The algorithm identifies the pair of adjacent bins that minimizes IV loss when merged:

$$\text{merge\_idx} = \arg \min_{i=0}^{k-2} (\text{IV}_i + \text{IV}_{i+1} - \text{IV}_{\text{merged}})$$

This greedy approach continues until  $k \leq \text{max\_bins}$ .

### Theoretical Foundations

- **Monotonicity as Stability Criterion:** Zeng (2014) proves that non-monotonic WoE patterns are unstable under population drift, leading to unreliable predictions when the data distribution shifts.
- **Regulatory Compliance:** Basel II/III validation requirements (BCBS, 2005) explicitly require monotonic relationships between risk drivers and probability of default for IRB models.
- **Information Preservation:** While enforcing monotonicity reduces model flexibility, Mironchyk & Tchistiakov (2017) demonstrate that the IV loss is typically < 5% compared to unconstrained binning for real credit portfolios.

### Comparison with Related Methods

Method	Monotonicity	Enforcement	Use Case
MOB	Guaranteed	During optimization	Regulatory scorecards
MBLP	Target	Iterative post-process	General credit models
MDLP	Optional	Post-hoc merging	Exploratory analysis
LDB	Optional	Post-hoc merging	Research/prototyping

### Computational Complexity

- Sorting:  $O(N \log N)$
- Pre-binning:  $O(N)$
- Rare bin merging:  $O(k^2 \times I_{\text{rare}})$  where  $I_{\text{rare}}$  is the number of rare bins
- Monotonicity enforcement:  $O(k^2 \times I_{\text{mono}})$  where  $I_{\text{mono}}$  is the number of violations (worst case:  $O(k)$ )
- Bin reduction:  $O(k \times (k_{\text{initial}} - \text{max\_bins}))$
- Total:  $O(N \log N + k^2 \times \text{max\_iterations})$

For typical credit scoring datasets ( $N \sim 10^5$ ,  $k \sim 5$ ), runtime is dominated by sorting. Pathological cases (e.g., perfectly alternating WoE values) may require  $O(k^2)$  merges.

### Value

A list containing:

- id** Integer vector of bin identifiers (1-based indexing).
- bin** Character vector of bin intervals in the format "[lower;upper)". The first bin starts with -Inf and the last bin ends with +Inf.
- woe** Numeric vector of Weight of Evidence values for each bin. Guaranteed to be monotonic (either non-decreasing or non-increasing).

- iv** Numeric vector of Information Value contributions for each bin.
- count** Integer vector of total observations in each bin.
- count\_pos** Integer vector of positive class (target = 1) counts per bin.
- count\_neg** Integer vector of negative class (target = 0) counts per bin.
- event\_rate** Numeric vector of event rates (proportion of positives) per bin.
- cutpoints** Numeric vector of cutpoints defining bin boundaries (excluding -Inf and +Inf). These are the upper bounds of bins 1 to k-1.
- total\_iv** Numeric scalar representing the total Information Value (sum of all bin IVs).
- converged** Logical flag indicating whether the algorithm converged within **max\_iterations**. FALSE indicates the iteration limit was reached during rare bin merging or monotonicity enforcement.
- iterations** Integer count of iterations performed across all optimization phases (rare bin merging + monotonicity enforcement + bin reduction).

### Author(s)

Lopes, J. E. (algorithm implementation based on Mironchyk & Tchistiakov, 2017)

### References

- Mironchyk, P., & Tchistiakov, V. (2017). "Monotone optimal binning algorithm for credit risk modeling". *Frontiers in Applied Mathematics and Statistics*, 3, 2.
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### See Also

[ob\\_numerical\\_mblp](#) for monotonicity-targeted binning with correlation-based direction detection,  
[ob\\_numerical\\_mdlp](#) for information-theoretic binning without monotonicity constraints.

### Examples

```
# Simulate non-monotonic credit scoring data
set.seed(42)
n <- 12000

# Create feature with inherent monotonic relationship + noise
feature <- c(
  rnorm(4000, mean = 600, sd = 50), # Low scores (high risk)
```

```
rnorm(5000, mean = 680, sd = 45), # Medium scores
rnorm(3000, mean = 740, sd = 35) # High scores (low risk)
)

target <- c(
  rbinom(4000, 1, 0.25), # 25% default
  rbinom(5000, 1, 0.10), # 10% default
  rbinom(3000, 1, 0.03) # 3% default
)

# Apply MOB
result <- ob_numerical_mob(
  feature = feature,
  target = target,
  min_bins = 2,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20
)

# Verify monotonicity
print(result$woe)
stopifnot(all(diff(result$woe) <= 1e-10)) # Non-increasing WoE

# Inspect binning quality
binning_table <- data.frame(
  Bin = result$bin,
  WoE = round(result$woe, 4),
  IV = round(result$iv, 4),
  Count = result$count,
  EventRate = round(result$event_rate, 4)
)
print(binning_table)

cat(sprintf("\nTotal IV: %.4f\n", result$total_iv))
cat(sprintf(
  "Converged: %s (iterations: %d)\n",
  result$converged, result$iterations
))

# Visualize monotonic pattern
oldpar <- par(mfrow = c(1, 2))

# WoE monotonicity
plot(result$woe,
  type = "b", col = "darkgreen", pch = 19, lwd = 2,
  xlab = "Bin", ylab = "WoE",
  main = "Guaranteed Monotonic WoE"
)
grid()

# Event rate vs WoE relationship
plot(result$event_rate, result$woe,
```

```

  pch = 19, col = "steelblue",
  xlab = "Event Rate", ylab = "WoE",
  main = "WoE vs Event Rate"
)
abline(lm(result$woe ~ result$event_rate), col = "red", lwd = 2)
grid()
par(oldpar)

```

---

### ob\_numerical\_mrbp

*Optimal Binning for Numerical Features using Monotonic Risk Binning with Likelihood Ratio Pre-binning*

---

## Description

Implements a greedy binning algorithm with monotonicity enforcement and **majority-vote direction detection**. *Important Note:* Despite the "Likelihood Ratio Pre-binning" designation in the name, the current implementation uses **equal-frequency pre-binning** without likelihood ratio statistics. The algorithm is functionally a variant of Monotonic Optimal Binning (MOB) with minor differences in merge strategies.

This method is suitable for credit scoring applications requiring monotonic WoE patterns, but users should be aware that it does not employ the statistical rigor implied by "Likelihood Ratio" in the name.

## Usage

```

ob_numerical_mrbp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  laplace_smoothing = 0.5
)

```

## Arguments

feature	Numeric vector of feature values to be binned. Missing values (NA) and infinite values are <b>not permitted</b> and will trigger an error (unlike other binning methods that issue warnings).
target	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as feature.

min_bins	Minimum number of bins to generate (default: 3). Must be at least 1. Acts as a hard constraint during monotonicity enforcement.
max_bins	Maximum number of bins to generate (default: 5). Must be greater than or equal to min_bins.
bin_cutoff	Minimum fraction of total observations required in each bin (default: 0.05). Bins with frequency below this threshold are merged. Must be in the range (0, 1).
max_n_prebins	Maximum number of pre-bins before optimization (default: 20). Must be at least equal to min_bins.
convergence_threshold	Convergence threshold (default: 1e-6). Currently used to check if WoE range is below threshold; primary stopping criterion is max_iterations.
max_iterations	Maximum number of iterations for bin merging and monotonicity enforcement (default: 1000). Prevents infinite loops.
laplace_smoothing	Laplace smoothing parameter for WoE calculation (default: 0.5). Must be non-negative.

## Details

### Algorithm Overview

The MRBLP algorithm executes in five phases:

#### Phase 1: Equal-Frequency Pre-binning

Initial bins are created by dividing sorted data into approximately equal-sized groups:

$$n_{\text{bin}} = \max \left( 1, \left\lfloor \frac{N}{\text{max\_n\_prebins}} \right\rfloor \right)$$

**Note:** Despite "Likelihood Ratio Pre-binning" in the name, no likelihood ratio statistics are computed. A true likelihood ratio approach would compute:

$$\text{LR}(c) = \prod_{x \leq c} \frac{P(x|y=1)}{P(x|y=0)} \times \prod_{x > c} \frac{P(x|y=1)}{P(x|y=0)}$$

and select cutpoints  $c$  that maximize  $|\log \text{LR}(c)|$ . This is **not implemented** in the current version.

#### Phase 2: Rare Bin Merging

Bins with total count below  $\text{bin\_cutoff} \times N$  are merged. The merge direction (left or right) is chosen to minimize IV loss:

$$\text{direction} = \arg \min_{d \in \{\text{left, right}\}} (\text{IV}_i + \text{IV}_{i+d} - \text{IV}_{\text{merged}})$$

#### Phase 3: Initial WoE/IV Calculation

Weight of Evidence for bin  $i$ :

$$\text{WoE}_i = \ln \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} \middle/ \frac{n_i^- + \alpha}{n^- + k\alpha} \right)$$

where  $\alpha$  = laplace\_smoothing and  $k$  is the number of bins.

#### Phase 4: Monotonicity Enforcement

The algorithm determines the desired monotonicity direction via **majority vote**:

$$\text{increasing} = \begin{cases} \text{TRUE} & \text{if } \#\{\text{WoE}_i > \text{WoE}_{i-1}\} \geq \#\{\text{WoE}_i < \text{WoE}_{i-1}\} \\ \text{FALSE} & \text{otherwise} \end{cases}$$

This differs from:

- **MOB**: Uses first two bins only ( $\text{WoE}[1] \geq \text{WoE}[0]$ )
- **MBLP**: Uses Pearson correlation between bin indices and WoE

Violations are detected as:

$$\text{violation} = \begin{cases} \text{WoE}_i < \text{WoE}_{i-1} & \text{if increasing} \\ \text{WoE}_i > \text{WoE}_{i-1} & \text{if decreasing} \end{cases}$$

Violating bins are merged iteratively until monotonicity is achieved or `min_bins` is reached.

#### Phase 5: Bin Count Reduction

If the number of bins exceeds `max_bins`, the algorithm merges bins with the **smallest absolute IV difference**:

$$\text{merge\_idx} = \arg \min_{i=0}^{k-2} |\text{IV}_i - \text{IV}_{i+1}|$$

**Critique**: This criterion assumes bins with similar IVs are redundant, which is not theoretically justified. A more rigorous approach (used in MBLP) minimizes IV loss **after merge**:

$$\Delta \text{IV} = \text{IV}_i + \text{IV}_{i+1} - \text{IV}_{\text{merged}}$$

### Theoretical Foundations

- **Monotonicity Enforcement**: Based on Zeng (2014), ensuring stability under data distribution shifts.
- **Likelihood Ratio (Theoretical)**: Neyman-Pearson lemma establishes likelihood ratio as the optimal test statistic for hypothesis testing. For binning, cutpoints maximizing LR would theoretically yield optimal class separation. **However, this is not implemented**.
- **Practical Equivalence**: The algorithm is functionally equivalent to MOB with minor differences in direction detection and merge strategies.

### Comparison with Related Methods

Method	Pre-binning	Direction Detection	Merge Criterion
MRBLP	Equal-frequency	Majority vote	Min IV difference
MOB	Equal-frequency	First two bins	Min IV loss
MBLP	Quantile-based	Pearson correlation	Min IV loss
MDLP	Equal-frequency	N/A (optional)	MDL cost

### Computational Complexity

Identical to MOB:  $O(N \log N + k^2 \times \text{max\_iterations})$

### When to Use MRBLP vs Alternatives

- **Use MRBLP:** If you specifically need majority-vote direction detection and can tolerate the non-standard merge criterion.
- **Use MOB:** For simplicity and slightly faster direction detection.
- **Use MBLP:** For more robust direction detection via correlation.
- **Use MDLP:** For information-theoretic optimality without mandatory monotonicity.

### Value

A list containing:

- id** Integer vector of bin identifiers (1-based indexing).
- bin** Character vector of bin intervals in the format "[lower;upper]".
- woe** Numeric vector of Weight of Evidence values. Guaranteed to be monotonic.
- iv** Numeric vector of Information Value contributions per bin.
- count** Integer vector of total observations per bin.
- count\_pos** Integer vector of positive class counts per bin.
- count\_neg** Integer vector of negative class counts per bin.
- event\_rate** Numeric vector of event rates per bin.
- cutpoints** Numeric vector of bin boundaries (excluding -Inf and +Inf).
- total\_iv** Total Information Value (sum of bin IVs).
- converged** Logical flag indicating convergence within `max_iterations`.
- iterations** Integer count of iterations performed.

### Author(s)

Lopes, J. E.

## References

- Neyman, J., & Pearson, E. S. (1933). "On the Problem of the Most Efficient Tests of Statistical Hypotheses". *Philosophical Transactions of the Royal Society A*, 231(694-706), 289-337. [Theoretical foundation for likelihood ratio, not implemented in code]
- Mironchyk, P., & Tchistiakov, V. (2017). "Monotone optimal binning algorithm for credit risk modeling". *Frontiers in Applied Mathematics and Statistics*, 3, 2.
- Zeng, G. (2014). "A Necessary Condition for a Good Binning Algorithm in Credit Scoring". *Applied Mathematical Sciences*, 8(65), 3229-3242.
- Siddiqi, N. (2006). *Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring*. Wiley.
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- Hosmer, D. W., Lemeshow, S., & Sturdivant, R. X. (2013). *Applied Logistic Regression* (3rd ed.). Wiley.

## See Also

[ob\\_numerical\\_mob](#) for the base monotonic binning algorithm, [ob\\_numerical\\_mblp](#) for correlation-based direction detection, [ob\\_numerical\\_md1p](#) for information-theoretic binning.

## Examples

```

# Simulate credit scoring data
set.seed(2024)
n <- 10000
feature <- c(
  rnorm(4000, mean = 620, sd = 50),
  rnorm(4000, mean = 690, sd = 45),
  rnorm(2000, mean = 740, sd = 35)
)
target <- c(
  rbinom(4000, 1, 0.20),
  rbinom(4000, 1, 0.10),
  rbinom(2000, 1, 0.04)
)

# Apply MRBLP
result <- ob_numerical_mrbp(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5
)

# Compare with MOB (should be very similar)
result_mob <- ob_numerical_mob(
  feature = feature,
  target = target,
  min_bins = 3,

```

```

    max_bins = 5
  )

# Compare results
data.frame(
  Method = c("MRBLP", "MOB"),
  N_Bins = c(length(result$woe), length(result_mob$woe)),
  Total_IV = c(result$total_iv, result_mob$total_iv),
  Iterations = c(result$iterations, result_mob$iterations)
)

```

---

ob\_numerical\_oslp

*Optimal Binning for Numerical Variables using Optimal Supervised Learning Partitioning*

---

## Description

Implements a greedy binning algorithm with quantile-based pre-binning and monotonicity enforcement. **Important Note:** Despite "Optimal Supervised Learning Partitioning" and "LP" in the name, the algorithm uses **greedy heuristics** without formal Linear Programming or convex optimization. The method is functionally equivalent to [ob\\_numerical\\_mrblp](#) with minor differences in pre-binning strategy and bin reduction criteria.

Users seeking true optimization-based binning should consider Mixed-Integer Programming (MIP) implementations (e.g., via `ompr` or `lpSolve` packages), though these scale poorly beyond  $N > 10,000$  observations.

## Usage

```

ob_numerical_oslp(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  laplace_smoothing = 0.5
)

```

## Arguments

feature	Numeric vector of feature values. Missing values (NA) and infinite values are <b>not permitted</b> and will trigger an error.
---------	---

target	Integer or numeric vector of binary target values (must contain only 0 and 1). Must have the same length as feature. Unlike other binning methods, OSLP internally uses double for target, allowing implicit conversion from integer.
min_bins	Minimum number of bins (default: 3). Must be at least 2.
max_bins	Maximum number of bins (default: 5). Must be greater than or equal to min_bins.
bin_cutoff	Minimum fraction of total observations per bin (default: 0.05). Must be in (0, 1).
max_n_prebins	Maximum number of pre-bins (default: 20). Must be at least equal to min_bins.
convergence_threshold	Convergence threshold for IV change (default: 1e-6).
max_iterations	Maximum iterations (default: 1000).
laplace_smoothing	Laplace smoothing parameter (default: 0.5). Must be non-negative.

## Details

### Algorithm Overview

OSLP executes in five phases:

#### Phase 1: Quantile-Based Pre-binning

Unlike equal-frequency methods that ensure balanced bin sizes, OSLP places cutpoints at quantiles of **unique feature values**:

$$\text{edge}_i = \text{unique\_vals} [\lfloor p_i \times (n_{\text{unique}} - 1) \rfloor]$$

where  $p_i = i/\text{max\_n\_prebins}$ .

**Critique:** If unique values are clustered (e.g., many observations at specific values), bins may have vastly different sizes, violating the equal-frequency principle that ensures statistical stability.

#### Phase 2: Rare Bin Merging

Bins with  $n_i/N < \text{bin\_cutoff}$  are merged. The merge direction minimizes IV loss:

$$\Delta \text{IV} = \text{IV}_i + \text{IV}_{i+d} - \text{IV}_{\text{merged}}$$

where  $d \in \{-1, +1\}$  (left or right neighbor).

#### Phase 3: Initial WoE/IV Calculation

Standard WoE with Laplace smoothing:

$$\text{WoE}_i = \ln \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} \middle/ \frac{n_i^- + \alpha}{n^- + k\alpha} \right)$$

#### Phase 4: Monotonicity Enforcement

Direction determined via majority vote (identical to MRBLP):

$$\text{increasing} = \begin{cases} \text{TRUE} & \text{if } \sum_i \mathbb{1}_{\{\text{WoE}_i > \text{WoE}_{i-1}\}} \geq \sum_i \mathbb{1}_{\{\text{WoE}_i < \text{WoE}_{i-1}\}} \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Violations are merged iteratively.

### Phase 5: Bin Count Reduction

If  $k > \text{max\_bins}$ , merge bins with the **smallest combined IV**:

$$\text{merge\_idx} = \arg \min_{i=0}^{k-2} (\text{IV}_i + \text{IV}_{i+1})$$

**Rationale:** Assumes bins with low total IV contribute least to predictive power. However, this ignores the interaction between bins; a low-IV bin may be essential for monotonicity or preventing gaps.

### Theoretical Foundations

Despite the name "Optimal Supervised Learning Partitioning", the algorithm lacks:

- **Global optimality guarantees:** Greedy merging is myopic
- **Formal loss function:** No explicit objective being minimized
- **LP formulation:** No constraint matrix, simplex solver, or dual variables

A true optimal partitioning approach would formulate the problem as:

$$\min_{\mathbf{z}, \mathbf{b}} \left\{ - \sum_{i=1}^k \text{IV}_i(\mathbf{b}) + \lambda k \right\}$$

subject to:

$$\sum_{j=1}^k z_{ij} = 1 \quad \forall i \in \{1, \dots, N\}$$

$$\text{WoE}_j \leq \text{WoE}_{j+1} \quad \forall j$$

$$z_{ij} \in \{0, 1\}, \quad b_j \in \mathbb{R}$$

where  $z_{ij}$  indicates observation  $i$  assigned to bin  $j$ , and  $\lambda$  is a complexity penalty. This requires MILP solvers (CPLEX, Gurobi) and is intractable for  $N > 10^4$ .

### Comparison with Related Methods

Method	Pre-binning	Direction	Merge (max_bins)	Target Type
OSLP	Quantile (unique vals)	Majority vote	Min (IV(i) + IV(i+1))	double
MRBLP	Equal-frequency	Majority vote	Min  IV(i) - IV(i+1)	int
MOB	Equal-frequency	First two bins	Min IV loss	int
MBLP	Quantile (data)	Correlation	Min IV loss	int

### When to Use OSLP

- **Use OSLP:** Never. Use MBLP or MOB instead for better pre-binning and merge strategies.
- **Use MBLP:** For robust direction detection via correlation.
- **Use MDLP:** For information-theoretic stopping criteria.
- **Use True LP:** For small datasets ( $N < 1000$ ) where global optimality is critical and computational cost is acceptable.

**Value**

A list containing:

- id** Integer bin identifiers (1-based).
- bin** Character bin intervals "[lower;upper)".
- woe** Numeric WoE values (guaranteed monotonic).
- iv** Numeric IV contributions per bin.
- count** Integer total observations per bin.
- count\_pos** Integer positive class counts.
- count\_neg** Integer negative class counts.
- event\_rate** Numeric event rates.
- cutpoints** Numeric bin boundaries (excluding  $\pm\text{Inf}$ ).
- total\_iv** Total Information Value.
- converged** Logical convergence flag.
- iterations** Integer iteration count.

**Author(s)**

Lopes, J. E.

**References**

- Mironchyk, P., & Tchistiakov, V. (2017). "Monotone optimal binning algorithm for credit risk modeling". *Frontiers in Applied Mathematics and Statistics*, 3, 2.
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**See Also**

[ob\\_numerical\\_mrblp](#) for nearly identical algorithm with better pre-binning, [ob\\_numerical\\_mb1p](#) for correlation-based direction detection, [ob\\_numerical\\_md1p](#) for information-theoretic optimality.

**Examples**

```
set.seed(123)
n <- 5000
feature <- c(
  rnorm(2000, 600, 50),
  rnorm(2000, 680, 40),
  rnorm(1000, 740, 30)
```

```

)
target <- c(
  rbinom(2000, 1, 0.25),
  rbinom(2000, 1, 0.10),
  rbinom(1000, 1, 0.03)
)

result <- ob_numerical_oslp(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5
)

print(result$woe)
print(result$total_iv)

# Compare with MRBLP (should be nearly identical)
result_mrblp <- ob_numerical_mrblp(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5
)

data.frame(
  Method = c("OSLP", "MRBLP"),
  Total_IV = c(result$total_iv, result_mrblp$total_iv),
  N_Bins = c(length(result$woe), length(result_mrblp$woe))
)

```

---

ob_numerical_sketch	<i>Optimal Binning for Numerical Variables using Sketch-based Algorithm</i>
---------------------	---

---

## Description

Implements optimal binning using the **\*\*KLL Sketch\*\*** (Karnin, Lang, Liberty, 2016), a probabilistic data structure for quantile approximation in data streams. This is the **only method in the package** that uses a fundamentally different algorithmic approach (streaming algorithms) compared to batch processing methods (MOB, MDLP, etc.).

The sketch-based approach enables:

- **Sublinear space complexity:**  $O(k \log N)$  vs  $O(N)$  for batch methods
- **Single-pass processing:** Suitable for streaming data
- **Provable approximation guarantees:** Quantile error  $\epsilon \approx O(1/k)$

The method combines KLL Sketch for candidate generation with either Dynamic Programming (for small  $N \leq 50$ ) or greedy IV-based selection (for larger datasets), followed by monotonicity enforcement via the Pool Adjacent Violators Algorithm (PAVA).

## Usage

```
ob_numerical_sketch(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  monotonic = TRUE,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  sketch_k = 200
)
```

## Arguments

feature	Numeric vector of feature values. Missing values (NA) are <b>not permitted</b> and will trigger an error. Infinite values (Inf, -Inf) and NaN are also not allowed.
target	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as feature. Missing values are not permitted.
min_bins	Minimum number of bins (default: 3). Must be at least 2.
max_bins	Maximum number of bins (default: 5). Must be $\geq \text{min\_bins}$ .
bin_cutoff	Minimum fraction of total observations per bin (default: 0.05). Must be in (0, 1). Bins with fewer observations will be merged with neighbors.
max_n_prebins	Maximum number of pre-bins to generate from quantiles (default: 20). This parameter controls the initial granularity of binning candidates. Higher values provide more flexibility but increase computational cost.
monotonic	Logical flag to enforce WoE monotonicity (default: TRUE). Uses PAVA (Pool Adjacent Violators Algorithm) for enforcement. Direction (increasing/ decreasing) is automatically detected from the data.
convergence_threshold	Convergence threshold for IV change (default: 1e-6). Optimization stops when the change in total IV between iterations falls below this value.
max_iterations	Maximum iterations for bin optimization (default: 1000). Prevents infinite loops in the optimization process.
sketch_k	Integer parameter controlling sketch accuracy (default: 200). Larger values improve quantile precision but increase memory usage. <b>Approximation error:</b> $\epsilon \approx 1/k$ (200 $\rightarrow$ 0.5% error). <b>Valid range:</b> [10, 1000]. Typical values: 50 (fast), 200 (balanced), 500 (precise).

## Details

### Algorithm Overview

The sketch-based binning algorithm executes in four phases:

#### Phase 1: KLL Sketch Construction

The KLL Sketch maintains a compressed, multi-level representation of the data distribution:

$$\text{Sketch} = \{\text{Compactor}_0, \text{Compactor}_1, \dots, \text{Compactor}_L\}$$

where each  $\text{Compactor}_\ell$  stores items with weight  $2^\ell$ . When a compactor exceeds capacity  $k$  (controlled by `sketch_k`), it is compacted.

**Theoretical Guarantees** (Karnin et al., 2016):

For a quantile  $q$  with estimated value  $\hat{q}$ :

$$|\text{rank}(\hat{q}) - q \cdot N| \leq \epsilon \cdot N$$

where  $\epsilon \approx O(1/k)$  and space complexity is  $O(k \log(N/k))$ .

#### Phase 2: Candidate Extraction

Approximately 40 quantiles are extracted from the sketch using a non-uniform grid with higher resolution in distribution tails.

#### Phase 3: Optimal Cutpoint Selection

For small datasets ( $N \leq 50$ ), Dynamic Programming maximizes total IV. For larger datasets, a greedy IV-based selection is used.

#### Phase 4: Bin Refinement

Bins are refined through frequency constraint enforcement, monotonicity enforcement (if requested), and bin count optimization to minimize IV loss.

### Computational Complexity

- **Time:**  $O(N \log k + N \times C + k^2 \times I)$
- **Space:**  $O(k \log N)$  for large  $N$

### When to Use Sketch-based Binning

- **Use:** Large datasets ( $N > 10^6$ ) with memory constraints or streaming data
- **Avoid:** Small datasets ( $N < 1000$ ) where approximation error may dominate

### Value

A list of class `c("OptimalBinningSketch", "OptimalBinning")` containing:

- id** Numeric vector of bin identifiers (1-based indexing).
- bin\_lower** Numeric vector of lower bin boundaries (inclusive).
- bin\_upper** Numeric vector of upper bin boundaries (inclusive for last bin, exclusive for others).
- woe** Numeric vector of Weight of Evidence values. Monotonic if `monotonic = TRUE`.

- iv** Numeric vector of Information Value contributions per bin.
- count** Integer vector of total observations per bin.
- count\_pos** Integer vector of positive class (target = 1) counts per bin.
- count\_neg** Integer vector of negative class (target = 0) counts per bin.
- cutpoints** Numeric vector of bin split points (length = number of bins - 1). These are the internal boundaries between bins.
- converged** Logical flag indicating whether optimization converged.
- iterations** Integer number of optimization iterations performed.

### Author(s)

Lopes, J. E.

### References

- Karnin, Z., Lang, K., & Liberty, E. (2016). "Optimal Quantile Approximation in Streams". *Proceedings of the 57th Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, 71-78. [doi:10.1109/FOCS.2016.20](https://doi.org/10.1109/FOCS.2016.20)
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- Siddiqi, N. (2006). *Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring*. Wiley. [doi:10.1002/9781119201731](https://doi.org/10.1002/9781119201731)

### See Also

[ob\\_numerical\\_mdlp](#), [ob\\_numerical\\_mb1p](#)

### Examples

```
# Example 1: Basic usage with simulated data
set.seed(123)
feature <- rnorm(500, mean = 100, sd = 20)
target <- rbinom(500, 1, prob = plogis((feature - 100) / 20))

result <- ob_numerical_sketch(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5
)

# Display results
print(data.frame(
  Bin = result$id,
  Count = result$count,
  WoE = round(result$woe, 4),
```

```

IV = round(result$iv, 4)
))

# Example 2: Comparing different sketch_k values
set.seed(456)
x <- rnorm(1000, 50, 15)
y <- rbinom(1000, 1, prob = 0.3)

result_k50 <- ob_numerical_sketch(x, y, sketch_k = 50)
result_k200 <- ob_numerical_sketch(x, y, sketch_k = 200)

cat("K=50 IV:", sum(result_k50$iv), "\n")
cat("K=200 IV:", sum(result_k200$iv), "\n")

```

---

ob\_numerical\_ubsd

*Optimal Binning for Numerical Variables using Unsupervised Binning with Standard Deviation*

---

## Description

Implements a **hybrid binning algorithm** that initializes bins using **unsupervised statistical properties** (mean and standard deviation of the feature) and refines them through **supervised optimization** using Weight of Evidence (WoE) and Information Value (IV).

**Important Clarification:** Despite "Unsupervised" in the name, this method is **predominantly supervised**. The unsupervised component is limited to the initial bin creation step (~1% of the algorithm). All subsequent refinement (merge, monotonicity enforcement, bin count adjustment) uses the target variable extensively.

The statistical initialization via  $\mu \pm k\sigma$  provides a data-driven starting point that may be advantageous for approximately normal distributions, but offers no guarantees for skewed or multimodal data.

## Usage

```

ob_numerical_ubsd(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  convergence_threshold = 1e-06,
  max_iterations = 1000,
  laplace_smoothing = 0.5
)

```

## Arguments

feature	Numeric vector of feature values. Missing values (NA) and infinite values are <b>not permitted</b> and will trigger an error.
target	Integer or numeric vector of binary target values (must contain only 0 and 1). Must have the same length as feature.
min_bins	Minimum number of bins (default: 3). Must be at least 2.
max_bins	Maximum number of bins (default: 5). Must be $\geq$ min_bins.
bin_cutoff	Minimum fraction of total observations per bin (default: 0.05). Must be in (0, 1).
max_n_prebins	Maximum number of pre-bins before optimization (default: 20). Must be at least equal to min_bins.
convergence_threshold	Convergence threshold for IV change (default: 1e-6).
max_iterations	Maximum iterations for optimization (default: 1000).
laplace_smoothing	Laplace smoothing parameter (default: 0.5). Must be non-negative.

## Details

### Algorithm Overview

UBSD executes in six phases:

#### Phase 1: Statistical Initialization (UNSUPERVISED)

Initial bin edges are created by combining two approaches:

##### 1. Standard deviation-based cutpoints:

$$\{\mu - 2\sigma, \mu - \sigma, \mu, \mu + \sigma, \mu + 2\sigma\}$$

where  $\mu$  is the sample mean and  $\sigma$  is the sample standard deviation (with Bessel correction:  $N - 1$  divisor).

##### 2. Equal-width cutpoints:

$$\left\{ x_{\min} + i \times \frac{x_{\max} - x_{\min}}{\max\_n\_prebins} \right\}_{i=1}^{\max\_n\_prebins-1}$$

The union of these two sets is taken, sorted, and limited to max\_n\_prebins edges (plus  $-\infty$  and  $+\infty$  boundaries).

**Rationale:** For approximately normal distributions,  $\mu \pm k\sigma$  cutpoints align with natural quantiles:

- $\mu - 2\sigma$  to  $\mu + 2\sigma$  captures ~95% of data (68-95-99.7 rule)
- Equal-width ensures coverage of entire range

**Limitation:** For skewed distributions (e.g., log-normal),  $\mu - 2\sigma$  may fall outside the data range, creating empty bins.

**Special Case:** If  $\sigma < \epsilon$  (feature is nearly constant), fallback to pure equal-width binning.

### Phase 2: Observation Assignment

Each observation is assigned to a bin via linear search:

$$\text{bin}(x_i) = \min\{j : x_i > \text{lower}_j \wedge x_i \leq \text{upper}_j\}$$

Counts are accumulated: count, count\_pos, count\_neg.

### Phase 3: Rare Bin Merging (SUPERVISED)

Bins with  $\text{count} < \text{bin\_cutoff} \times N$  are merged with adjacent bins. Merge direction is chosen to minimize IV loss:

$$\text{direction} = \arg \min_{d \in \{\text{left, right}\}} (\text{IV}_i + \text{IV}_{i+d})$$

This is a **supervised** step (uses IV computed from target).

### Phase 4: WoE/IV Calculation (SUPERVISED)

Weight of Evidence with Laplace smoothing:

$$\text{WoE}_i = \ln \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} \middle/ \frac{n_i^- + \alpha}{n^- + k\alpha} \right)$$

Information Value:

$$\text{IV}_i = \left( \frac{n_i^+ + \alpha}{n^+ + k\alpha} - \frac{n_i^- + \alpha}{n^- + k\alpha} \right) \times \text{WoE}_i$$

### Phase 5: Monotonicity Enforcement (SUPERVISED)

Direction is auto-detected via majority vote:

$$\text{increasing} = \begin{cases} \text{TRUE} & \text{if } \sum_i \mathbb{1}_{\{\text{WoE}_i > \text{WoE}_{i-1}\}} \geq \sum_i \mathbb{1}_{\{\text{WoE}_i < \text{WoE}_{i-1}\}} \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Violations are resolved via PAVA (Pool Adjacent Violators Algorithm).

### Phase 6: Bin Count Adjustment (SUPERVISED)

If  $k > \text{max\_bins}$ , bins are merged to minimize IV loss:

$$\text{merge\_idx} = \arg \min_{i=0}^{k-2} (\text{IV}_i + \text{IV}_{i+1})$$

**Convergence Criterion:**

$$|\text{IV}_{\text{total}}^{(t)} - \text{IV}_{\text{total}}^{(t-1)}| < \text{convergence\_threshold}$$

### Comparison with Related Methods

Method	Initialization	Truly Unsupervised?	Best For
UBSD	$\mu \pm k\sigma$ + equal-width	No (1 pct unsup)	Normal distributions
MOB/MRBLP	Equal-frequency	No (0 pct unsup)	General use

MDLP	Equal-frequency	No (0 pct unsup)	Information theory
Sketch	KLL Sketch quantiles	No (0 pct unsup)	Streaming data

## When to Use UBSD

- **Use UBSD:** If you have prior knowledge that the feature is approximately normally distributed and want bins aligned with standard deviations (e.g., for interpretability: "2 standard deviations below mean").
- **Avoid UBSD:** For skewed distributions (use MDLP or MOB), for multimodal distributions (use LDB), or when you need provable optimality (use Sketch for quantile guarantees).
- **Alternative:** For true unsupervised binning (no target), use `cut()` with `breaks = "Sturges"` or `"FD"` (Freedman-Diaconis).

## Computational Complexity

Identical to MOB/MRBLP:  $O(N + k^2 \times \text{max\_iterations})$

## Value

A list containing:

- id** Integer bin identifiers (1-based).
- bin** Character bin intervals "[lower;upper]".
- woe** Numeric WoE values (monotonic after enforcement).
- iv** Numeric IV contributions per bin.
- count** Integer total observations per bin.
- count\_pos** Integer positive class counts.
- count\_neg** Integer negative class counts.
- event\_rate** Numeric event rates per bin.
- cutpoints** Numeric bin boundaries (excluding  $\pm\infty$ ).
- total\_iv** Total Information Value.
- converged** Logical convergence flag.
- iterations** Integer iteration count.

## Author(s)

Lopes, J. E.

## References

- Sturges, H. A. (1926). "The Choice of a Class Interval". *Journal of the American Statistical Association*, 21(153), 65-66.
- Scott, D. W. (1979). "On optimal and data-based histograms". *Biometrika*, 66(3), 605-610.
- Freedman, D., & Diaconis, P. (1981). "On the histogram as a density estimator: L2 theory". *Zeitschrift fuer Wahrscheinlichkeitstheorie*, 57(4), 453-476.

- Thomas, L. C. (2009). *Consumer Credit Models: Pricing, Profit, and Portfolios*. Oxford University Press.
- Zeng, G. (2014). "A Necessary Condition for a Good Binning Algorithm in Credit Scoring". *Applied Mathematical Sciences*, 8(65), 3229-3242.
- Siddiqi, N. (2006). *Credit Risk Scorecards*. Wiley.

## See Also

[ob\\_numerical\\_mdlp](#) for information-theoretic binning, [ob\\_numerical\\_mob](#) for pure supervised binning, [cut](#) for true unsupervised binning.

## Examples

```
# Simulate normally distributed credit scores
set.seed(123)
n <- 5000

# Feature: Normally distributed FICO scores
feature <- rnorm(n, mean = 680, sd = 60)

# Target: Logistic relationship with score
prob_default <- 1 / (1 + exp((feature - 680) / 30))
target <- rbinom(n, 1, prob_default)

# Apply UBSD
result <- ob_numerical_ubsd(
  feature = feature,
  target = target,
  min_bins = 3,
  max_bins = 5
)

# Compare with MDLP (should be similar for normal data)
result_mdlp <- ob_numerical_mdlp(feature, target)

data.frame(
  Method = c("UBSD", "MDLP"),
  N_Bins = c(length(result$woe), length(result_mdlp$woe)),
  Total_IV = c(result$total_iv, result_mdlp$total_iv)
)
```

## Description

Implements a supervised binning algorithm that uses Information Gain (Entropy) to identify the most informative initial split points, followed by a bottom-up merging process to satisfy constraints (minimum frequency, monotonicity, max bins).

Although historically referred to as "Unsupervised Decision Trees" in some contexts, this method is strictly **\*\*supervised\*\*** (uses target variable) and operates **\*\*bottom-up\*\*** after an initial entropy-based selection of cutpoints. It is particularly effective when the relationship between feature and target is non-linear but highly informative in specific regions.

## Usage

```
ob_numerical_udt(
  feature,
  target,
  min_bins = 3,
  max_bins = 5,
  bin_cutoff = 0.05,
  max_n_prebins = 20,
  laplace_smoothing = 0.5,
  monotonicity_direction = "none",
  convergence_threshold = 1e-06,
  max_iterations = 1000
)
```

## Arguments

<code>feature</code>	Numeric vector of feature values. Missing values (NA) are handled by placing them in a separate bin. Infinite values are treated as valid numeric extremes or placed in the missing bin if they represent errors.
<code>target</code>	Integer vector of binary target values (must contain only 0 and 1). Must have the same length as <code>feature</code> .
<code>min_bins</code>	Minimum number of bins (default: 3). Must be at least 2.
<code>max_bins</code>	Maximum number of bins (default: 5). Must be greater than or equal to <code>min_bins</code> .
<code>bin_cutoff</code>	Minimum fraction of total observations per bin (default: 0.05). Bins below this threshold are merged based on Event Rate similarity.
<code>max_n_prebins</code>	Maximum number of pre-bins (default: 20). The algorithm will select the top <code>max_n_prebins</code> cutpoints with highest Information Gain. <b>Performance Note:</b> High values (>50) may significantly slow down processing for large datasets due to the $O(N^2)$ nature of cutpoint selection.
<code>laplace_smoothing</code>	Laplace smoothing parameter (default: 0.5) for WoE calculation.
<code>monotonicity_direction</code>	String specifying monotonicity constraint: <ul style="list-style-type: none"> <li>• "none" (default): No monotonicity enforcement.</li> <li>• "increasing": WoE must be non-decreasing.</li> <li>• "decreasing": WoE must be non-increasing.</li> </ul>

- "auto": Automatically determined by Pearson correlation.

convergence\_threshold  
Convergence threshold for IV optimization (default: 1e-6).

max\_iterations Maximum iterations for optimization loop (default: 1000).

## Details

### Algorithm Overview

The UDT algorithm executes in four phases:

#### Phase 1: Entropy-Based Pre-binning

The algorithm evaluates every possible cutpoint  $c$  (midpoints between sorted unique values) using Information Gain (IG):

$$IG(S, c) = H(S) - \left( \frac{|S_L|}{|S|} H(S_L) + \frac{|S_R|}{|S|} H(S_R) \right)$$

The top `max_n_prebins` cutpoints with the highest IG are selected to form the initial bins. This ensures that the starting bins capture the most discriminative regions of the feature space.

#### Phase 2: Rare Bin Merging

Bins with frequency  $< \text{bin\_cutoff}$  are merged. The merge partner is chosen to minimize the difference in Event Rates:

$$\text{merge\_idx} = \arg \min_{j \in \{i-1, i+1\}} |ER_i - ER_j|$$

This differs from IV-based methods and aims to preserve local risk probability smoothness.

#### Phase 3: Monotonicity Enforcement

If requested, monotonicity is enforced by iteratively merging bins that violate the specified direction ("increasing", "decreasing", or "auto"). Auto-direction is determined by the sign of the Pearson correlation between feature and target.

#### Phase 4: Constraint Satisfaction

If  $k > \text{max\_bins}$ , bins are merged minimizing IV loss until the constraint is met.

### Warning on Complexity

The pre-binning phase evaluates Information Gain for *all* unique values. For continuous features with many unique values (e.g.,  $N > 10,000$ ), this step can be computationally intensive ( $O(N^2)$ ). Consider rounding or using [ob\\_numerical\\_sketch](#) for very large datasets.

## Value

A list containing:

- id** Integer bin identifiers (1-based).
- bin** Character bin intervals "(lower;upper]".
- woe** Numeric WoE values.
- iv** Numeric IV contributions.
- event\_rate** Numeric event rates.

**count** Integer total observations.  
**count\_pos** Integer positive class counts.  
**count\_neg** Integer negative class counts.  
**cutpoints** Numeric bin boundaries.  
**total\_iv** Total Information Value.  
**gini** Gini index (2\*AUC - 1) calculated on the binned data.  
**ks** Kolmogorov-Smirnov statistic calculated on the binned data.  
**converged** Logical convergence flag.  
**iterations** Integer iteration count.

### Author(s)

Lopes, J. E.

### References

- Quinlan, J. R. (1986). "Induction of Decision Trees". *Machine Learning*, 1(1), 81-106.
- Fayyad, U. M., & Irani, K. B. (1992). "On the Handling of Continuous-Valued Attributes in Decision Tree Generation". *Machine Learning*, 8, 87-102.
- Liu, H., et al. (2002). "Discretization: An Enabling Technique". *Data Mining and Knowledge Discovery*, 6(4), 393-423.

### See Also

[ob\\_numerical\\_mdlp](#) for a pure MDL-based approach, [ob\\_numerical\\_sketch](#) for fast approximation on large data.

---

ob\_preprocess

*Data Preprocessor for Optimal Binning*

---

### Description

Prepares features for optimal binning by handling missing values and optionally detecting/treating outliers. Supports both numerical and categorical variables with configurable preprocessing strategies.

### Usage

```
ob_preprocess(
  feature,
  target,
  num_miss_value = -999,
  char_miss_value = "N/A",
  outlier_method = "iqr",
```

```

    outlier_process = FALSE,
    preprocess = "both",
    iqr_k = 1.5,
    zscore_threshold = 3,
    grubbs_alpha = 0.05
)

```

## Arguments

feature	Vector (numeric, character, or factor) to be preprocessed. Type is automatically detected.
target	Numeric or integer vector of binary target values (0/1). Must have the same length as feature. Used for validation but not directly in preprocessing.
num_miss_value	Numeric value to replace missing (NA) values in numerical features (default: -999.0). Choose a value outside the expected range of the feature.
char_miss_value	Character string to replace missing (NA) values in categorical features (default: "N/A").
outlier_method	Character string specifying the outlier detection method for numerical features (default: "iqr"). Options: <ul style="list-style-type: none"> <li>• "iqr": Interquartile Range method. Outliers are values <math>&lt; Q_1 - k \times IQR</math> or <math>&gt; Q_3 + k \times IQR</math>.</li> <li>• "zscore": Z-score method. Outliers are values with <math> z  &gt;</math> threshold where <math>z = (x - \mu)/\sigma</math>.</li> <li>• "grubbs": Grubbs' test for outliers (iterative). Removes the most extreme value if it exceeds the critical G-statistic at significance level grubbs_alpha.</li> </ul>
outlier_process	Logical flag to enable outlier detection and treatment (default: FALSE). Only applies to numerical features.
preprocess	Character vector specifying output components (default: "both"): <ul style="list-style-type: none"> <li>• "feature": Return preprocessed feature data only.</li> <li>• "report": Return preprocessing report only (summary statistics, counts).</li> <li>• "both": Return both preprocessed data and report.</li> </ul>
iqr_k	Multiplier for the IQR method (default: 1.5). Larger values are more conservative (fewer outliers). Common values: 1.5 (standard), 3.0 (extreme).
zscore_threshold	Z-score threshold for outlier detection (default: 3.0). Values with $ z  >$ threshold are considered outliers.
grubbs_alpha	Significance level for Grubbs' test (default: 0.05). Lower values are more conservative (fewer outliers detected).

## Details

### Preprocessing Pipeline:

1. **Type Detection:** Automatically classifies feature as numeric or categorical based on R type.

2. **Missing Value Handling:** Replaces NA with num\_miss\_value (numeric) or char\_miss\_value (categorical).
3. **Outlier Detection** (if outlier\_process = TRUE for numeric):
  - **IQR Method:** Caps outliers at boundaries  $[Q_1 - k \times IQR, Q_3 + k \times IQR]$ .
  - **Z-score Method:** Caps outliers at  $[\mu - t \times \sigma, \mu + t \times \sigma]$ .
  - **Grubbs' Test:** Iteratively removes the most extreme value if  $G = \frac{\max |x_i - \bar{x}|}{s} > G_{\text{critical}}$ .
4. **Summary Calculation:** Computes statistics before and after preprocessing for validation.

### Outlier Treatment Strategies:

- IQR and Z-score: **Winsorization** (capping at boundaries).
- Grubbs: **Removal** (replaced with num\_miss\_value).

### Use Cases:

- **Before binning:** Stabilize binning algorithms by removing extreme values that could create singleton bins.
- **Data quality audit:** Identify features with excessive missingness or outliers.
- **Model deployment:** Ensure test data undergoes identical preprocessing as training data.

### Value

A list with up to two elements (depending on preprocess):

**preprocess** Data frame with columns:

- feature: Original feature values.
- feature\_preprocessed: Preprocessed feature values (NAs replaced, outliers capped or removed).

**report** Data frame with one row containing:

- variable\_type: "numeric" or "categorical".
- missing\_count: Number of NA values replaced.
- outlier\_count: Number of outliers detected (numeric only, NA for categorical).
- original\_stats: String representation of summary statistics before preprocessing (min, Q1, median, mean, Q3, max for numeric).
- preprocessed\_stats: Summary statistics after preprocessing.

### References

- Grubbs, F. E. (1950). "Sample Criteria for Testing Outlying Observations". *Annals of Mathematical Statistics*, 21(1), 27-58.
- Tukey, J. W. (1977). *Exploratory Data Analysis*. Addison-Wesley. [IQR method]

## Examples

```
# Numerical feature with outliers
set.seed(123)
feature_num <- c(rnorm(95, 50, 10), NA, NA, 200, -100, 250)
target <- sample(0:1, 100, replace = TRUE)

# Preprocess with IQR outlier detection
result_iqr <- ob_preprocess(
  feature = feature_num,
  target = target,
  outlier_process = TRUE,
  outlier_method = "iqr",
  iqr_k = 1.5
)

print(result_iqr$report)
# Shows: missing_count = 2, outlier_count = 3

# Categorical feature
feature_cat <- c(rep("A", 30), rep("B", 40), rep("C", 28), NA, NA)
target_cat <- sample(0:1, 100, replace = TRUE)

result_cat <- ob_preprocess(
  feature = feature_cat,
  target = target_cat,
  char_miss_value = "Missing"
)

# Compare original vs preprocessed
head(result_cat$preprocess)
# Shows NA replaced with "Missing"

# Return only report (no data)
result_report <- ob_preprocess(
  feature = feature_num,
  target = target,
  preprocess = "report",
  outlier_process = TRUE
)

# Grubbs' test (most conservative)
result_grubbs <- ob_preprocess(
  feature = feature_num,
  target = target,
  outlier_process = TRUE,
  outlier_method = "grubbs",
  grubbs_alpha = 0.01 # Very strict
)
```

plot.obwoe

*Plot Method for obwoe Objects*

## Description

Creates publication-quality visualizations of optimal binning results. Supports multiple plot types including IV ranking charts, WoE profiles, and bin distribution plots. All plots follow credit scoring visualization conventions.

## Usage

```
## S3 method for class 'obwoe'
plot(
  x,
  type = c("iv", "woe", "bins"),
  feature = NULL,
  top_n = 15,
  show_threshold = TRUE,
  ...
)
```

## Arguments

x	An object of class "obwoe".
type	Character string specifying the plot type: "iv" Information Value ranking bar chart (default) "woe" Weight of Evidence profile for selected features "bins" Bin distribution (count and event rate)
feature	Character vector of feature names to plot (for "woe" and "bins" types). If NULL, uses top 6 features by IV.
top_n	Integer. For "iv" type, number of top features to display. Default is 15. Set to NULL to display all.
show_threshold	Logical. For "iv" type, draw horizontal lines at IV thresholds (0.02, 0.10, 0.30)? Default is TRUE.
...	Additional arguments passed to base plotting functions.

## Details

### Plot Types:

**IV Ranking** (type = "iv"): Horizontal bar chart showing features ranked by Information Value. Colors indicate predictive power classification:

- Gray: IV < 0.02 (Unpredictive)
- Yellow: 0.02 <= IV < 0.10 (Weak)
- Orange: 0.10 <= IV < 0.30 (Medium)

- Green:  $0.30 \leq IV < 0.50$  (Strong)
- Red:  $IV \geq 0.50$  (Suspicious)

**WoE Profile** (type = "woe"): Bar chart showing Weight of Evidence values for each bin. Positive WoE indicates higher-than-average event rate; negative WoE indicates lower-than-average event rate. Monotonic WoE patterns are generally preferred for interpretability.

**Bin Distribution** (type = "bins"): Dual-axis plot showing observation counts (bars) and event rates (line). Useful for diagnosing bin quality and class imbalance.

## Value

Invisibly returns NULL. Called for side effect (plotting).

## References

Thomas, L. C., Edelman, D. B., & Crook, J. N. (2002). Credit Scoring and Its Applications. *SIAM Monographs on Mathematical Modeling and Computation*. doi:10.1137/1.9780898718317

## See Also

[obwoe](#), [summary.obwoe](#).

## Examples

```
set.seed(42)
df <- data.frame(
  age = rnorm(500, 40, 15),
  income = rgamma(500, 2, 0.0001),
  score = rnorm(500, 600, 100),
  target = rbinom(500, 1, 0.2)
)
result <- obwoe(df, target = "target")

# IV ranking chart
plot(result, type = "iv")

# WoE profile for specific feature
plot(result, type = "woe", feature = "age")

# Bin distribution
plot(result, type = "bins", feature = "income")
```

---

plot.obwoe\_gains *Plot Gains Table*

---

### Description

Visualizes gains table metrics including cumulative capture curves, KS plot, and lift chart.

### Usage

```
## S3 method for class 'obwoe_gains'
plot(x, type = c("cumulative", "ks", "lift", "woe_iv"), ...)
```

### Arguments

x	An object of class "obwoe_gains".
type	Character string: "cumulative" (default), "ks", "lift", or "woe_iv".
...	Additional arguments passed to plotting functions.

### Value

Invisibly returns NULL.

---

prep.step\_obwoe *Prepare the Optimal Binning Step*

---

### Description

Fits the optimal binning models on training data. This method is called by `prep` and should not be invoked directly.

### Usage

```
## S3 method for class 'step_obwoe'
prep(x, training, info = NULL, ...)
```

### Arguments

x	A step_obwoe object.
training	A tibble or data frame containing the training data.
info	A tibble with column metadata from the recipe.
...	Additional arguments (currently unused).

### Value

A trained step\_obwoe object with `binning_results` populated.

---

`print.obwoe`*Print Method for obwoe Objects*

---

**Description**

Displays a concise summary of optimal binning results, including the number of successfully processed features and top predictors ranked by Information Value.

**Usage**

```
## S3 method for class 'obwoe'  
print(x, ...)
```

**Arguments**

<code>x</code>	An object of class "obwoe".
<code>...</code>	Additional arguments (currently ignored).

**Value**

Invisibly returns `x`.

**See Also**

[summary.obwoe](#) for detailed statistics, [plot.obwoe](#) for visualization.

---

`print.step_obwoe`*Print Method for step\_obwoe*

---

**Description**

Prints a concise summary of the `step_obwoe` object.

**Usage**

```
## S3 method for class 'step_obwoe'  
print(x, width = max(20L, options()$width - 30L), ...)
```

**Arguments**

<code>x</code>	A <code>step_obwoe</code> object.
<code>width</code>	Maximum width for printing term names.
<code>...</code>	Additional arguments (currently unused).

**Value**

Invisibly returns `x`.

---

required\_pkgs.step\_obwoe

*Required Packages for step\_obwoe*

---

## Description

Lists the packages required to execute the step\_obwoe transformation.

## Usage

```
## S3 method for class 'step_obwoe'
required_pkgs(x, ...)
```

## Arguments

x	A step_obwoe object.
...	Additional arguments (currently unused).

## Value

A character vector of package names.

---

step\_obwoe

*Optimal Binning and WoE Transformation Step*

---

## Description

step\_obwoe() creates a *specification* of a recipe step that discretizes predictor variables using one of 28 state-of-the-art optimal binning algorithms and transforms them into Weight of Evidence (WoE) values. This step fully integrates the **OptimalBinningWoE** package into the **tidymodels** framework, supporting supervised discretization for both binary and multinomial classification targets with extensive hyperparameter tuning capabilities.

## Usage

```
step_obwoe(
  recipe,
  ...,
  role = "predictor",
  trained = FALSE,
  outcome = NULL,
  algorithm = "auto",
  min_bins = 2L,
  max_bins = 10L,
  bin_cutoff = 0.05,
```

```

  output = c("woe", "bin", "both"),
  suffix_woe = "_woe",
  suffix_bin = "_bin",
  na_woe = 0,
  control = list(),
  binning_results = NULL,
  skip = FALSE,
  id = recipes::rand_id("obwoe")
)

```

## Arguments

recipe	A recipe object. The step will be added to the sequence of operations for this recipe.
...	One or more selector functions to choose variables for this step. See <a href="#">selections</a> for available selectors. Common choices include <code>all_predictors()</code> , <code>all_numeric_predictors()</code> , or <code>all_nominal_predictors()</code> . Ensure the selected variables are compatible with the chosen algorithm (e.g., do not apply "mdlp" to categorical data).
role	For variables created by this step, what role should they have? Default is "predictor".
trained	A logical indicating whether the step has been trained (fitted). This should not be set manually.
outcome	A character string specifying the name of the binary or multinomial response variable. This argument is <b>required</b> as all binning algorithms are supervised. The outcome must exist in the training data provided to <code>prep()</code> . The outcome should be encoded as a factor (standard for <code>tidymodels</code> classification) or as integers 0/1 for binary, 0/1/2/... for multinomial.
algorithm	Character string specifying the binning algorithm to use. Use "auto" (default) for automatic selection based on target type: "jedi" for binary targets, "jedi_mwoe" for multinomial.  Available algorithms are organized by supported feature types: <b>Universal (numerical and categorical):</b> "auto", "jedi", "jedi_mwoe", "cm", "dp", "dmiv", "fetb", "mob", "sketch", "udt" <b>Numerical only:</b> "bb", "ewb", "fast_mdlp", "ir", "kmb", "ldb", "lpdb", "mblp", "mdlp", "mrblp", "oslp", "ubsd" <b>Categorical only:</b> "gmb", "ivb", "mba", "milp", "sab", "sblp", "swb" This parameter is tunable with <code>tune()</code> .
min_bins	Integer specifying the minimum number of bins to create. Must be at least 2. Default is 2. This parameter is tunable with <code>tune()</code> .
max_bins	Integer specifying the maximum number of bins to create. Must be greater than or equal to <code>min_bins</code> . Default is 10. This parameter is tunable with <code>tune()</code> .
bin_cutoff	Numeric value between 0 and 1 (exclusive) specifying the minimum proportion of total observations that each bin must contain. Bins with fewer observations are merged with adjacent bins. This serves as a regularization mechanism to prevent overfitting and ensure statistical stability of WoE estimates. Default is 0.05 (5%). This parameter is tunable with <code>tune()</code> .

output	Character string specifying the transformation output format: "woe" Replaces the original variable with WoE values (default). This is the standard choice for logistic regression scorecards. "bin" Replaces the original variable with bin labels (character). Useful for tree-based models or exploratory analysis. "both" Keeps the original column and adds two new columns with suffixes _woe and _bin. Useful for model comparison or audit trails.
suffix_woe	Character string suffix appended to create WoE column names when output = "both". Default is "_woe".
suffix_bin	Character string suffix appended to create bin column names when output = "both". Default is "_bin".
na_woe	Numeric value to assign to observations that cannot be mapped to a bin during bake(). This includes missing values (NA) and unseen categories not present in the training data. Default is 0, which represents neutral evidence (neither good nor bad).
control	A named list of additional control parameters passed to <a href="#">control.obwoe</a> . These provide fine-grained control over algorithm behavior such as convergence thresholds and maximum pre-bins. Parameters specified directly in step_obwoe() (e.g., bin_cutoff) take precedence over values in this list.
binning_results	Internal storage for fitted binning models after prep(). Do not set manually.
skip	Logical. Should this step be skipped when bake() is called on new data? Default is FALSE. Setting to TRUE is rarely needed for WoE transformations but may be useful in specialized workflows.
id	A unique character string to identify this step. If not provided, a random identifier is generated.

## Details

### Weight of Evidence Transformation:

Weight of Evidence (WoE) is a supervised encoding technique that transforms categorical and continuous variables into a scale that measures the predictive strength of each value or bin relative to the target variable. For a bin  $i$ , the WoE is defined as:

$$WoE_i = \ln \left( \frac{\text{Distribution of Events}_i}{\text{Distribution of Non-Events}_i} \right)$$

Positive WoE values indicate the bin has a higher proportion of events (e.g., defaults) than the overall population, while negative values indicate lower risk.

### Algorithm Selection Strategy:

The algorithm parameter provides access to 28 binning algorithms:

- Use algorithm = "auto" (default) for automatic selection: "jedi" for binary targets, "jedi\_mwoe" for multinomial.
- Use algorithm = "mob" (Monotonic Optimal Binning) when monotonic WoE trends are required for regulatory compliance (Basel/IFRS 9).

- Use `algorithm = "mdlp"` for entropy-based discretization of numerical variables (requires `all_numeric_predictors()`).
- Use `algorithm = "dp"` (Dynamic Programming) for exact optimal solutions when computational cost is acceptable.

If an incompatible algorithm is applied to a variable (e.g., `"mdlp"` on a factor), the step will issue a warning during `prep()` and skip that variable, leaving it untransformed.

### Handling New Data:

During `bake()`, observations are mapped to bins learned during `prep()`:

- **Numerical variables:** Values are assigned to bins based on the learned cutpoints using interval notation.
- **Categorical variables:** Categories are matched to their corresponding bins. Categories not seen during training receive the `na_woe` value.
- **Missing values:** Always receive the `na_woe` value.

### Tuning with tune:

This step is fully compatible with the `tune` package. The following parameters support `tune()`:

- `algorithm`: See [obwoe\\_algorithm](#).
- `min_bins`: See [obwoe\\_min\\_bins](#).
- `max_bins`: See [obwoe\\_max\\_bins](#).
- `bin_cutoff`: See [obwoe\\_bin\\_cutoff](#).

### Case Weights:

This step does not currently support case weights. All observations are treated with equal weight during binning optimization.

### Value

An updated `recipe` object with the new step appended.

### References

Siddiqi, N. (2006). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. *John Wiley & Sons*. doi:[10.1002/9781119201731](https://doi.org/10.1002/9781119201731)

Thomas, L. C., Edelman, D. B., & Crook, J. N. (2002). Credit Scoring and Its Applications. *SIAM Monographs on Mathematical Modeling and Computation*. doi:[10.1137/1.9780898718317](https://doi.org/10.1137/1.9780898718317)

Navas-Palencia, G. (2020). Optimal Binning: Mathematical Programming Formulation and Solution Approach. *Expert Systems with Applications*, 158, 113508. doi:[10.1016/j.eswa.2020.113508](https://doi.org/10.1016/j.eswa.2020.113508)

### See Also

[obwoe](#) for the underlying binning engine, [control.obwoe](#) for advanced control parameters, [obwoe\\_algorithm](#), [obwoe\\_min\\_bins](#), [obwoe\\_max\\_bins](#), [obwoe\\_bin\\_cutoff](#) for tuning parameter definitions, [recipe](#), [prep](#), [bake](#) for the `tidymodels` recipe framework.

## Examples

```

library(recipes)

# Simulated credit data
set.seed(123)
credit_data <- data.frame(
  age = rnorm(500, 45, 12),
  income = exp(rnorm(500, 10, 0.6)),
  employment = sample(c("Employed", "Self-Employed", "Unemployed"),
  500,
  replace = TRUE, prob = c(0.7, 0.2, 0.1)
),
  education = factor(c("HighSchool", "Bachelor", "Master", "PhD")[
  sample(1:4, 500, replace = TRUE, prob = c(0.3, 0.4, 0.2, 0.1))
]),
  default = factor(rbinom(500, 1, 0.15),
  levels = c(0, 1),
  labels = c("No", "Yes")
)
)
)

# Example 1: Basic usage with automatic algorithm selection
rec_basic <- recipe(default ~ ., data = credit_data) %>%
  step_obwoe(all_predictors(), outcome = "default")

rec_prep <- prep(rec_basic)
baked_data <- bake(rec_prep, new_data = NULL)
head(baked_data)

# View binning details
tidy(rec_prep, number = 1)

# Example 2: Numerical-only algorithm on numeric predictors
rec_mdlp <- recipe(default ~ age + income, data = credit_data) %>%
  step_obwoe(all_numeric_predictors(),
  outcome = "default",
  algorithm = "mdlp",
  min_bins = 3,
  max_bins = 6
)

# Example 3: Output both bins and WoE
rec_both <- recipe(default ~ age, data = credit_data) %>%
  step_obwoe(age,
  outcome = "default",
  output = "both"
)

baked_both <- bake(prep(rec_both), new_data = NULL)
names(baked_both)
# Contains: default, age, age_woe, age_bin

```

```

# Example 4: Custom control parameters
rec_custom <- recipe(default ~ ., data = credit_data) %>%
  step_obwoe(all_predictors(),
             outcome = "default",
             algorithm = "mob",
             bin_cutoff = 0.03,
             control = list(
               max_n_prebins = 30,
               convergence_threshold = 1e-8
             )
  )

# Example 5: Tuning specification (for use with tune package)
# rec_tune <- recipe(default ~ ., data = credit_data) %>%
#   step_obwoe(all_predictors(),
#             outcome = "default",
#             algorithm = tune(),
#             min_bins = tune(),
#             max_bins = tune())

```

---

summary.obwoe

*Summary Method for obwoe Objects*

---

## Description

Generates comprehensive summary statistics for optimal binning results, including predictive power classification based on established IV thresholds (Siddiqi, 2006), aggregate metrics, and feature-level diagnostics.

## Usage

```
## S3 method for class 'obwoe'
summary(object, sort_by = "iv", decreasing = TRUE, ...)
```

## Arguments

object	An object of class "obwoe".
sort_by	Character string specifying the column to sort by. Options: "iv" (default), "n_bins", "feature".
decreasing	Logical. Sort in decreasing order? Default is TRUE for IV, FALSE for feature names.
...	Additional arguments (currently ignored).

## Details

### IV Classification Thresholds:

Following Siddiqi (2006), features are classified by predictive power:

Classification	IV Range
Unpredictive	< 0.02
Weak	0.02 - 0.10
Medium	0.10 - 0.30
Strong	0.30 - 0.50
Suspicious	> 0.50

Features with IV > 0.50 should be examined for data leakage or overfitting, as such high values are rarely observed in practice.

## Value

An S3 object of class "summary.obwoe" containing:

`feature_summary` Data frame with per-feature statistics including IV classification (Unpredictive/Weak/Medium/Strong/Suspicious)

`aggregate` Named list of aggregate statistics:

- `n_features` Total features processed
- `n_successful` Features without errors
- `n_errors` Features with errors
- `total_iv_sum` Sum of all feature IVs
- `mean_iv` Mean IV across features
- `median_iv` Median IV across features
- `mean_bins` Mean number of bins
- `iv_range` Min and max IV values
- `iv_distribution` Table of IV classification counts
- `target` Target column name
- `target_type` Target type (binary/multinomial)

## References

Siddiqi, N. (2006). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. *John Wiley & Sons*. doi:10.1002/9781119201731

## See Also

[obwoe](#) for the main binning function, [print.obwoe](#), [plot.obwoe](#).

## Examples

```
set.seed(42)
df <- data.frame(
  x1 = rnorm(500), x2 = rnorm(500), x3 = rnorm(500),
  target = rbinom(500, 1, 0.2)
)
result <- obwoe(df, target = "target")
summary(result)
```

---

tidy.step\_obwoe

*Tidy Method for step\_obwoe*

---

## Description

Returns a tibble with information about the binning transformation. For trained steps, returns one row per bin per feature, including bin labels, WoE values, and IV contributions. For untrained steps, returns a placeholder tibble.

## Usage

```
## S3 method for class 'step_obwoe'
tidy(x, ...)
```

## Arguments

- x A step\_obwoe object.
- ... Additional arguments (currently unused).

## Value

A tibble with columns:

- terms** Character. Feature name.
- bin** Character. Bin label or interval.
- woe** Numeric. Weight of Evidence value for the bin.
- iv** Numeric. Information Value contribution of the bin.
- id** Character. Step identifier.

---

`tunable.step_obwoe` *Tunable Parameters for step\_obwoe*

---

### Description

Returns information about which parameters of `step_obwoe` can be tuned using the `tune` package.

### Usage

```
## S3 method for class 'step_obwoe'  
tunable(x, ...)
```

### Arguments

<code>x</code>	A <code>step_obwoe</code> object.
<code>...</code>	Additional arguments (currently unused).

### Value

A tibble describing tunable parameters.

# Index

.categorical\_only\_algorithms, 3  
.numerical\_only\_algorithms, 4  
.universal\_algorithms, 4  
.valid\_algorithms, 5

bake, 5, 199  
bake.step\_obwoe, 5

control.obwoe, 6, 13, 16, 23, 198, 199  
cut, 185

fit\_logistic\_regression, 8

ob\_apply\_woe\_cat, 16, 30, 32  
ob\_apply\_woe\_num, 16, 31  
ob\_categorical\_cm, 30, 33, 39, 44, 48, 54, 61, 66, 84  
ob\_categorical\_dmv, 37, 44, 48  
ob\_categorical\_dp, 41, 48, 54, 61, 66, 76, 84  
ob\_categorical\_fetb, 46, 54  
ob\_categorical\_gmb, 51, 61, 66  
ob\_categorical\_ivb, 16, 58, 66, 76  
ob\_categorical\_jedi, 16, 63, 76, 84  
ob\_categorical\_jedi\_mwoe, 72  
ob\_categorical\_mba, 80  
ob\_categorical\_milp, 88  
ob\_categorical\_mob, 90  
ob\_categorical\_sab, 93  
ob\_categorical\_sbfp, 96  
ob\_categorical\_sketch, 99  
ob\_categorical\_swb, 103  
ob\_categorical\_udt, 106  
ob\_cutpoints\_cat, 109  
ob\_cutpoints\_num, 111  
ob\_gains\_table, 112, 114, 115  
ob\_gains\_table\_feature, 114  
ob\_numerical\_bb, 115, 125  
ob\_numerical\_cm, 118, 125, 127, 128, 131, 133, 138, 142  
ob\_numerical\_dmv, 121

ob\_numerical\_dp, 123, 128, 131, 136  
ob\_numerical\_ewb, 126, 142  
ob\_numerical\_fast\_mdlp, 129  
ob\_numerical\_fetb, 132  
ob\_numerical\_ir, 134, 138  
ob\_numerical\_jedi, 16, 136, 140, 149  
ob\_numerical\_jedi\_mwoe, 138  
ob\_numerical\_kmb, 141, 149  
ob\_numerical\_ldb, 143, 154, 160  
ob\_numerical\_lpdb, 147  
ob\_numerical\_mblp, 150, 160, 166, 172, 176, 180  
ob\_numerical\_mdlp, 16, 31, 32, 146, 154, 155, 166, 172, 176, 180, 185, 188  
ob\_numerical\_mob, 31, 32, 146, 162, 172, 185  
ob\_numerical\_mrblp, 168, 173, 176  
ob\_numerical\_oslp, 173  
ob\_numerical\_sketch, 177, 187, 188  
ob\_numerical\_ubs, 181  
ob\_numerical\_udt, 185  
ob\_preprocess, 188  
obcorr, 10  
obwoe, 6, 7, 12, 19–21, 24, 26, 193, 199, 202  
obwoe\_algorithm, 19, 199  
obwoe\_algorithms, 16, 20  
obwoe\_apply, 20, 24, 26  
obwoe\_bin\_cutoff, 22, 199  
obwoe\_gains, 23  
obwoe\_max\_bins, 28, 29, 199  
obwoe\_min\_bins, 28, 29, 199

plot.obwoe, 192, 195, 202  
plot.obwoe\_gains, 26, 194  
prep, 194, 199  
prep.step\_obwoe, 194  
print.obwoe, 16, 195, 202  
print.step\_obwoe, 195

recipe, 199  
required\_pkgs.step\_obwoe, 196

selections, [197](#)  
step\_obwoe, [19](#), [22](#), [23](#), [28](#), [29](#), [196](#)  
summary.obwoe, [21](#), [193](#), [195](#), [201](#)  
  
tidy.step\_obwoe, [203](#)  
tunable.step\_obwoe, [204](#)