Package ‘ciuupi’

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Type Package

Title Confidence Intervals Utilizing Uncertain Prior Information

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Description Computes a confidence interval for a specified linear combination of the regression parameters in a linear regression model with iid normal errors with known variance when there is uncertain prior information that a distinct specified linear combination of the regression parameters takes a given value. This confidence interval, found by numerical nonlinear constrained optimization, has the required minimum coverage and utilizes this uncertain prior information through desirable expected length properties.
This confidence interval has the following three practical applications.
Firstly, if the error variance has been accurately estimated from previous data then it may be treated as being effectively known. Secondly, for sufficiently large (dimension of the response vector) minus (dimension of regression parameter vector), greater than or equal to 30 (say), if we replace the assumed known value of the error variance by its usual estimator in the formula for the confidence interval then the resulting interval has, to a very good approximation, the same coverage probability and expected length properties as when the error variance is known. Thirdly, some more complicated models can be approximated by the linear regression model with error variance known when certain unknown parameters are replaced by estimates. This confidence interval is described in
Mainzer, R. and Kabaila, P.
(2019) <doi:10.32614/RJ-2019-026>, and is a member of the family of confidence intervals proposed by Kabaila, P. and Giri, K. (2009)
<doi:10.1016/j.jspi.2009.03.018>.

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`acX_to_rho` *Computes the known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$ from the p-vectors a and c and the design matrix X*

Description

Computes the known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$. This correlation is computed from the p-vectors a and c and the $n \times p$ design matrix X, with linearly independent columns, using the formula $\rho = a^\top (X^\top X)^{-1} c / (v_\theta \, v_\tau)^{1/2}$, where $v_\theta = a^\top (X^\top X)^{-1} a$ and $v_\tau = c^\top (X^\top X)^{-1} c$.

Usage

```
acX_to_rho(a, c, X)
```

Arguments

`a`	The p-vector a that specifies the parameter of interest $\theta = a^\top \beta$
`c`	The p-vector c used in the specification of the parameter $\tau = c^\top \beta - t$. The uncertain prior information is that $\tau = 0$
`X`	The $n \times p$ design matrix X, with linearly independent columns

Value

The known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$.

Examples

```
a <- c(0, 2, 0, -2)
c <- c(0, 0, 0, 1)
x1 <- c(-1, 1, -1, 1)
x2 <- c(-1, -1, 1, 1)
X <- cbind(rep(1, 4), x1, x2, x1*x2)
rho <- acX_to_rho(a, c, X)
print(rho)
```

`bs.list.example` *The list that specifies the CIUUPI for the example*

Description

In this example, the dataset described in Table 7.5 of Box et al. (1963) is used. The design matrix X is specified by the command `X <- cbind(rep(1,4), c(-1, 1, -1, 1), c(-1, -1, 1, 1), c(1, -1, -1, 1))`. A description of the parameter of interest is given in Discussion 5.8, p.3426 of Kabaila and Giri (2009). The parameter of interest is $\theta = a^\top \beta$, where the column vector a is specified by the command `a <- c(0, 2, 0, -2)`. For this example, we have uncertain prior information that $\tau = c^\top \beta = 0$, where the column vector c is specified by the command `c <- c(0, 0, 0, 1)`. The known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$ is computed using the command `rho <- acX_to_rho(a, c, X)`. The desired minimum coverage probability of the CIUUPI is $1 - \alpha$, where $\alpha = 0.05$, which is specified by the command `alpha <- 0.05`. The CIUUPI is determined by α and ρ and is found using the command `bs.list.example <- bs_ciuupi(alpha, rho)`, which takes about 5 minutes to run.

Usage

```
bs.list.example
```

Format

An object of class `list` of length 8.

References

Box, G.E.P., Connor, L.R., Cousins, W.R., Davies, O.L., Hinsworth, F.R., Sillitto, G.P. (1963) The Design and Analysis of Industrial Experiments, 2nd edition, reprinted. Oliver and Boyd, London.

Kabaila, P. and Giri, K. (2009) Confidence intervals in regression utilizing prior information. Journal of Statistical Planning and Inference, 139, 3419 - 3429.

`bsspline` *Evaluate the functions b and s at* `x`

Description

Evaluate the functions b and s, as specified by `bsvec`, `alpha`, `d`, `n.ints` and `natural`, at `x`.

Usage

```
bsspline(x, bsvec, alpha, d, n.ints, natural)
```

Arguments

- `x` A value or vector of values at which the functions b and s are to be evaluated
- `bsvec` The $(2q-1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

  where q=ceiling(d/0.75) and $h = d/q$. This vector specifies the CIUUPI, for all possible values of the random error variance and the observed response vector
- `alpha` The desired minimum coverage probability is $1 - \alpha$
- `d` The functions b and s are specified by cubic splines on the interval $[-d, d]$
- `n.ints` The number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.
- `natural` Equal to 1 (default) if the b and s functions are obtained by natural cubic spline interpolation or 0 if obtained by clamped cubic spline interpolation

Value

A data frame containing `x` and the corresponding values of the functions b and s.

Examples

```
x <- seq(0, 8, by = 1)
alpha <- bs.list.example$alpha
natural <- bs.list.example$natural
d <- bs.list.example$d
n.ints <- bs.list.example$n.ints
bsvec <- bs.list.example$bsvec
bs <- bsspline(x, bsvec, alpha, d, n.ints, natural)
```

`bs_ciuupi` *Computes the the functions b and s that specify the CIUUPI for all possible values of σ and the observed response vector*

Description

Chooses the positive number d and the positive integer q, sets $h = d/q$, and then computes the $(2q-1)$-vector $\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big)$ that determines, via cubic spline interpolation, the functions b and s which specify the confidence interval for θ that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and the observed response vector. To an excellent approximation, this confidence interval has minimum coverage probability $1 - \alpha$.

Usage

```
bs_ciuupi(alpha, rho, natural = 1)
```

Arguments

`alpha`	The desired minimum coverage probability is $1 - \alpha$
`rho`	The known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$
`natural`	Equal to 1 (default) if the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation in an interval $[-d, d]$, where d is computed by `bs_ciuupi` using a specified function of `alpha` and `rho`

Details

Suppose that

$$y = X\beta + \varepsilon$$

where y is a random n-vector of responses, X is a known n by p matrix with linearly independent columns, β is an unknown parameter p-vector and ε is the random error with components that are iid normally distributed with zero mean and known variance σ^2. The parameter of interest is $\theta = a^\top \beta$. Also let $\tau = c^\top \beta - t$, where a and c are specified linearly independent vectors and t is a specified number. The uncertain prior information is that $\tau = 0$.

Let `rho` denote the known correlation between the $\widehat{\theta}$ and $\widehat{\tau}$. We can compute `rho` from given values of a, c and X using the function `acX_to_rho`.

The confidence interval for θ, with minimum coverage probability $1-$`alpha`, that utilizes the uncertain prior information that $\tau = 0$ belongs to a class of confidence intervals indexed by the functions b and s. The function b is an odd continuous function and the function s is an even continuous function. In addition, $b(x) = 0$ and $s(x)$ is equal to the $1-$`alpha`$/2$ quantile of the standard normal distribution for all $|x| \geq d$, where d is a given positive number. Extensive numerical explorations have been used to find a formula (in terms of `alpha` and `rho`) for a 'goldilocks' value of d that is neither too large nor too small. Then let q=ceiling(d/0.75) and $h = d/q$. The values of the functions b and s in the interval $[-d, d]$ are specified by the $(2q-1)$-vector

$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big)$.

The values of $b(kh)$ and $s(kh)$ for $k = -q, ..., q$ are deduced from this vector using the assumptions made about the functions b and s. The values of $b(x)$ and $s(x)$ for any x in the interval $[-d, d]$ are then found using cube spline interpolation using the values of $b(kh)$ and $s(kh)$ for $k = -q, ..., q$. For `natural=1` (default) this is 'natural' cubic spline interpolation and for `natural=0` this is 'clamped' cubic spline interpolation.

The vector $\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big)$ is found by numerical nonlinear constrained optimization so that the confidence interval has minimum coverage probability 1−`alpha` and utilizes the uncertain prior information through its desirable expected length properties. This optimization is performed using the `slsqp` function in the `nloptr` package.

Value

A list with the following components.

alpha, rho, natural: the inputs

d: a 'goldilocks' value of d that is not too large and not too small

n.ints: number of equal-length consecutive intervals whose union is $[0, d]$, this is the same as q

lambda.star: the computed value of λ^*

bsvec: the vector $\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big)$ that determines the functions b and s that specify the CIUUPI for all possible values of σ and observed response vector

comp.time: the computation time in seconds

Examples

```
alpha <- 0.05
rho <- - 1 / sqrt(2)
bs.list <- bs_ciuupi(alpha, rho)
```

`ciuupi_observed_value` *For given observed response vector y, compute the confidence interval that utilizes the uncertain prior information (CIUUPI)*

Description

If σ is provided then, for given observed response vector y, compute the confidence interval, with minimum coverage probability $1 - \alpha$, for the parameter $\theta = a^\top \beta$ that utilizes the uncertain prior information that the parameter $\tau = c^\top \beta - t$ (specified by the vector c and the number t) takes the value 0. If σ is not provided then, as long as $n - p \geq 30$, replace σ by its estimate to compute an approximation to the CIUUPI for θ.

Usage

```
ciuupi_observed_value(a, c, X, alpha, bs.list, t, y, sig = NULL)
```

Arguments

`a`	The p-vector a that specifies the parameter of interest $\theta = a^\top \beta$
`c`	The p-vector c used in the specification of the parameter $\tau = c^\top \beta - t$. The uncertain prior information is that $\tau = 0$
`X`	The $n \times p$ design matrix X, with linearly independent columns
`alpha`	$1 - \alpha$ is the desired minimum coverage probability of the confidence interval for θ
`bs.list`	A list that includes the following components: natural, d, q and the vector bsvec (b(h),...,b((q-1)h), s(0),s(h)...,s((q-1)h)), where h=d/q, that specifies the CIUUPI for all possible values of the random error variance and the observed response vector
`t`	The number t used to specify the parameter $\tau = c^\top \beta - t$. The uncertain prior information is that $\tau = 0$
`y`	The n-vector of observed responses
`sig`	Standard deviation of the random error. If a value is not specified then, provided that $n - p \geq 30$, `sig` is estimated from the data.

Details

Suppose that

$$y = X\beta + \varepsilon$$

where y is a random n-vector of responses, X is a known $n \times p$ matrix with linearly independent columns, β is an unknown parameter p-vector and ε has components that are iid normally distributed with zero mean and known variance. Suppose that $\theta =$`a`$^\top$ β is the parameter of interest, where `a` is a specified vector. Let $\tau =$`c`$^\top \beta -$`t`, where `c` is a specified vector, `t` is a specified number and `a` and `c` are linearly independent vectors. Also suppose that we have uncertain prior information that $\tau = 0$. For given observed response vector `y` and a design matrix `X`, `ciuupi_observed_value` computes the confidence interval, with minimum coverage probability $1-$`alpha`, for θ that utilizes the uncertain prior information that $\tau = 0$.

The example below is described in Discussion 5.8 on p.3426 of Kabaila and Giri (2009). This example is obtained by extracting a 2×2 factorial data set from the 2^3 factorial data set described in Table 7.5 of Box et al. (1963).

Value

If σ is provided then a data frame of the lower and upper endpoints of the confidence interval, with minimum coverage probability $1-\alpha$, for the parameter θ that utilizes the uncertain prior information that $\tau = 0$. If σ is not provided then, as long as $n - p \geq 30$, a data frame of the lower and upper endpoints of an approximation to this confidence interval.

References

Box, G.E.P., Connor, L.R., Cousins, W.R., Davies, O.L., Hinsworth, F.R., Sillitto, G.P. (1963) The Design and Analysis of Industrial Experiments, 2nd edition, reprinted. Oliver and Boyd, London.

Kabaila, P. and Giri, K. (2009) Confidence intervals in regression utilizing prior information. Journal of Statistical Planning and Inference, 139, 3419 - 3429.

Examples

```
a <- c(0, 2, 0, -2)
c <- c(0, 0, 0, 1)
x1 <- c(-1, 1, -1, 1)
x2 <- c(-1, -1, 1, 1)
X <- cbind(rep(1, 4), x1, x2, x1*x2)
alpha <- 0.05
t <- 0
y <- c(87.2, 88.4, 86.7, 89.2)
sig <- 0.8
ciuupi_observed_value(a, c, X, alpha, bs.list.example, t, y, sig=sig)
```

---

ci_standard — *For given observed response vector y, compute the standard $1 - \alpha$ confidence interval*

Description

If σ is provided then compute the standard $1 - \alpha$ confidence interval for θ. If σ is not provided then, as long as $n - p \geq 30$, replace σ by its estimate to compute an approximate $1 - \alpha$ confidence interval for θ.

Usage

```
ci_standard(a, X, y, alpha, sig = NULL)
```

Arguments

a	The vector used to specify the parameter of interest $\theta = a^\top \beta$
X	The known $n \times p$ design matrix, with linearly independent columns
y	The n-vector of observed responses
alpha	$1 - \alpha$ is the coverage probability of the standard confidence interval
sig	Standard deviation of the random error. If a value is not specified then, provided that $n - p \geq 30$, `sig` is estimated from the data.

Details

Suppose that

$$y = X\beta + \varepsilon,$$

where y is a random n-vector of responses, X is a known $n \times p$ matrix with linearly independent columns, β is an unknown parameter p-vector, and $\varepsilon \sim N(0, \sigma^2 I)$, with σ^2 assumed known. Suppose that the parameter of interest is $\theta = a^\top \beta$. The R function `ci_standard` computes the standard $1 - \alpha$ confidence interval for θ.

The example below is described in Discussion 5.8 on p.3426 of Kabaila and Giri (2009). This example is obtained by extracting a 2×2 factorial data set from the 2^3 factorial data set described in Table 7.5 of Box et al. (1963).

Value

If σ is provided then a data frame of the lower and upper endpoints of the standard $1-\alpha$ confidence interval for θ. If σ is not provided then, as long as $n - p \geq 30$, a data frame of the lower and upper endpoints of an approximation to this confidence interval.

References

Box, G.E.P., Connor, L.R., Cousins, W.R., Davies, O.L., Hinsworth, F.R., Sillitto, G.P. (1963) The Design and Analysis of Industrial Experiments, 2nd edition, reprinted. Oliver and Boyd, London.

Kabaila, P. and Giri, K. (2009) Confidence intervals in regression utilizing prior information. Journal of Statistical Planning and Inference, 139, 3419 - 3429.

Examples

```
y <- c(87.2, 88.4, 86.7, 89.2)
x1 <- c(-1, 1, -1, 1)
x2 <- c(-1, -1, 1, 1)
X <- cbind(rep(1, 4), x1, x2, x1*x2)
a <- c(0, 2, 0, -2)
ci_standard(a, X, y, 0.05, sig = 0.8)
```

`cpciuupi` *Compute the coverage probability of the CIUUPI*

Description

Evaluate the coverage probability of the confidence interval that utilizes uncertain prior information (CIUUPI) at `gam`. The input `bs.list` determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector.

Usage

```
cpciuupi(gam, n.nodes, bs.list)
```

Arguments

`gam` A value of γ or vector of values of γ at which the coverage probability function is evaluated

`n.nodes` The number of nodes for the Gauss Legendre quadrature used for the evaluation of the coverage probability

`bs.list` A list that includes the following components.

`alpha`: $1-\alpha$ is the desired minimum coverage probability of the confidence interval

`rho`: The known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$

`natural`: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

`d`: the functions b and s are specified by cubic splines on the interval $[-d, d]$

`n.ints`: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.

`bsvec`: the $(2q-1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

The value(s) of the coverage probability of the CIUUPI at `gam`.

Examples

```
gam <- seq(0, 10, by = 0.2)
n.nodes <- 10
cp <- cpciuupi(gam, n.nodes, bs.list.example)
```

`plot_b` *Plot the graph of the odd function b used in the specification of the CIUUPI*

Description

The input `bs.list` determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector. The R function `plot_b` plots the graph of the odd function b.

Usage

```
plot_b(bs.list)
```

Arguments

`bs.list` A list that includes the following components.

`alpha`: the desired minimum coverage is $1 - \alpha$.

`rho`: the known correlation between $\widehat{\theta}$ and $\widehat{\tau}$. This correlation is computed from the p-vectors a and c and the $n \times p$ design matrix X using the formula $\rho = a^\top (X^\top X)^{-1} c / (v_\theta \, v_\tau)^{1/2}$, where $v_\theta = a^\top (X^\top X)^{-1} a$ and $v_\tau = c^\top (X^\top X)^{-1} c$.

`natural`: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

`d`: the functions b and s are specified by cubic splines on the interval $[-d, d]$

`n.ints`: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.

`bsvec`: the $(2q - 1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

A plot of the graph of the odd function b used in the specification of the CIUUPI.

Examples

```
plot_b(bs.list.example)
```

`plot_cp` *Plot the graph of the coverage probability of the CIUUPI*

Description

The input `bs.list` determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector. The coverage probability of the CIUUPI is an even function of the unknown parameter $\gamma = \tau \big/ \big(\sigma\, v_\tau^{1/2}\big)$. The R function `plot_cp` plots the graph of the coverage probability of the CIUUPI, as a function of $|\gamma|$. To provide a stringent assessment of this coverage probability, we use a fine equally-spaced grid `seq(0, (d+4), by = 0.01)` of values of γ and Gauss Legendre quadrature using 10 nodes in the relevant integrals. By contrast, for the computation of the CIUUPI, implemented in `bs_ciuupi`, we require that the coverage probability of this confidence interval is greater than or equal to $1 - \alpha$ for the equally-spaced grid `seq(0, (d+2), by = 0.05)` of values of γ and we use Gauss Legendre quadrature with 5 nodes in the relevant integrals.

Usage

```
plot_cp(bs.list)
```

Arguments

bs.list
: A list that includes the following components.

alpha: the desired minimum coverage is $1 - \alpha$.

rho: the known correlation between $\widehat{\theta}$ and $\widehat{\tau}$. This correlation is computed from the p-vectors a and c and the $n \times p$ design matrix X using the formula $\rho = a^\top (X^\top X)^{-1} c / (v_\theta \, v_\tau)^{1/2}$, where $v_\theta = a^\top (X^\top X)^{-1} a$ and $v_\tau = c^\top (X^\top X)^{-1} c$.

natural: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

d: the functions b and s are specified by cubic splines on the interval $[-d, d]$

n.ints: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of bsvec, n.ints is also called q.

bsvec: the $(2q - 1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

A plot of the graph of the coverage probability of the CIUUPI as a function of $|\gamma|$, where γ denotes the unknown parameter $\tau \big/ \big(\sigma \, v_\tau^{1/2}\big)$.

Examples

```
plot_cp(bs.list.example)
```

---

plot_s — *Plot the graph of the even function s used in the specification of the CIUUPI*

Description

The input bs.list determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector. The R function plot_s plots the graph of the odd function s.

Usage

```
plot_s(bs.list)
```

Arguments

`bs.list` A list that includes the following components.

`alpha`: the desired minimum coverage is $1 - \alpha$.

`rho`: the known correlation between $\widehat{\theta}$ and $\widehat{\tau}$. This correlation is computed from the p-vectors a and c and the $n \times p$ design matrix X using the formula $\rho = a^\top (X^\top X)^{-1} c / (v_\theta \, v_\tau)^{1/2}$, where $v_\theta = a^\top (X^\top X)^{-1} a$ and $v_\tau = c^\top (X^\top X)^{-1} c$.

`natural`: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

`d`: the functions b and s are specified by cubic splines on the interval $[-d, d]$

`n.ints`: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.

`bsvec`: the $(2q - 1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

A plot of the graph of the even function s used in the specification of the CIUUPI.

Examples

```
plot_s(bs.list.example)
```

`plot_squared_sel` *Plot the graph of the squared scaled expected length of the CIUUPI*

Description

The input `bs.list` determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector. The scaled expected length of the CIUUPI is an even function of the unknown parameter $\gamma = \tau \big/ \big(\sigma \, v_\tau^{1/2}\big)$. The R function `plot_squared_sel` plots the graph of the squared scaled expected length (i.e. squared SEL) of the CIUUPI, as a function of $|\gamma|$. To provide a stringent assessment of this squared SEL, we use a grid `seq(0, (d+4), by = 0.01)` of values of γ and Gauss Legendre quadrature with 10 nodes in the relevant integrals. By contrast, for the computation of the CIUUPI, implemented in `bs_ciuupi`, we use Gauss Legendre quadrature with 5 nodes in the relevant integrals.

Usage

```
plot_squared_sel(bs.list)
```

Arguments

`bs.list` A list that includes the following components.

`alpha`: the desired minimum coverage is $1 - \alpha$.

`rho`: the known correlation between $\widehat{\theta}$ and $\widehat{\tau}$. This correlation is computed from the p-vectors a and c and the $n \times p$ design matrix X using the formula $\rho = a^\top (X^\top X)^{-1} c / (v_\theta \, v_\tau)^{1/2}$, where $v_\theta = a^\top (X^\top X)^{-1} a$ and $v_\tau = c^\top (X^\top X)^{-1} c$.

`natural`: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

`d`: the functions b and s are specified by cubic splines on the interval $[-d, d]$

`n.ints`: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.

`bsvec`: the $(2q - 1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

A plot of the graph of the squared scaled expected length (i.e. squared SEL) of the CIUUPI as a function of $|\gamma|$, where γ denotes the unknown parameter $\tau / \big(\sigma \, v_\tau^{1/2}\big)$.

Examples

```
plot_squared_sel(bs.list.example)
```

`selciuupi` *Compute the scaled expected length of the CIUUPI*

Description

Evaluate the scaled expected length of the confidence interval that utilizes uncertain prior information (CIUUPI) at `gam`. This scaled expected length is defined to be the expected length of the CIUUPI divided by the expected length of the standard $1-\alpha$ confidence interval. The input `bs.list` determines the functions b and s that specify the confidence interval that utilizes the uncertain prior information (CIUUPI), for all possible values of σ and observed response vector.

Usage

```
selciuupi(gam, n.nodes, bs.list)
```

Arguments

`gam` A value of γ or vector of γ values at which the coverage probability function is evaluated

`n.nodes` The number of nodes for the Gauss Legendre quadrature used for the evaluation of the scaled expected length

`bs.list` A list that includes the following components.

`alpha`: $1 - \alpha$ is the desired minimum coverage probability of the confidence interval

`rho`: The known correlation ρ between $\widehat{\theta}$ and $\widehat{\tau}$

`natural`: 1 when the functions b and s are specified by natural cubic spline interpolation or 0 if these functions are specified by clamped cubic spline interpolation

`d`: the functions b and s are specified by cubic splines on the interval $[-d, d]$

`n.ints`: number of equal-length intervals in $[0, d]$, where the endpoints of these intervals specify the knots, belonging to $[0, d]$, of the cubic spline interpolations that specify the functions b and s. In the description of `bsvec`, `n.ints` is also called q.

`bsvec`: the $(2q - 1)$-vector

$$\big(b(h), ..., b((q-1)h), s(0), s(h)..., s((q-1)h)\big),$$

where q=ceiling(d/0.75) and $h = d/q$.

Value

The value(s) of the scaled expected length at `gam`.

Examples

```
gam <- seq(0, 10, by = 0.2)
n.nodes <- 10
sel <- selciuupi(gam, n.nodes, bs.list.example)
```

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